

## Final exam, 13 December 2016

All aids are allowed, except a computer-like device (including tablets and smartphones) and personal assistance. The exam consists of three questions/assignments that should all be answered. The weights for each of the three questions and also for each subquestion within a question are indicated; these weights total *50 points*. Note that questions and subquestions can be answered independently of each other. The duration of the exam is 3 hours.

Generally, **statistical models and methods should be specified**.

### Question 1. (*15 points*)

A study has been carried out at AVC by Dr. Etienne Côté on effects of a drug treatment in cats with a specific heart condition (left ventricular concentric hypertrophy). A total of 12 cats were recruited for the study among cats referred to the AVC Cardiology Service that satisfied certain criteria. Each cat had measurements taken of serum cardiac troponin-I (in the following abbreviated as sct-I) concentrations both prior to treatment and after a treatment period of at least 8 days. The sct-I concentrations were used as cardiac biomarkers to assess the effect of the treatment. In addition, prior to treatment an echocardiographic analysis for each cat was used to measure the degree to which the cat suffered from a specific heart condition (left ventricular outflow tract obstruction); these measurements are denoted as (blood) velocities. The values obtained for the 12 cats are presented in the table below.

cat no.	sct-I concentration			velocity
	pre	post	change	
1	0.07	0.00	0.07	1.1
2	4.03	0.19	3.84	1.8
3	0.29	0.05	0.24	1.4
4	0.08	0.04	0.04	1.0
5	0.05	0.00	0.05	1.1
6	0.26	0.09	0.17	4.3
7	0.00	0.00	0.00	0.9
8	0.00	0.00	0.00	1.0
9	0.00	0.00	0.00	1.1
10	0.06	0.04	0.02	1.5
11	0.21	0.04	0.17	1.1
12	0.00	0.00	0.00	1.2

Use the data of the table and any (not necessarily all) of the attached Minitab listings to answer the questions.

- a) (3 points) Carry out a (brief) descriptive analysis for the pre and post sct-I concentration variables. Focus on the characteristics of each distribution that you consider important for statistical analysis instead of giving a detailed enumeration of the values of different descriptive statistics.
- b) (4 points) One of the questions of primary interest in the study was how the cardiac biomarker sct-I responded to the treatment. Use a statistical test to assess whether the data show evidence of a treatment effect. State your statistical assumptions, hypotheses and conclusions carefully.
- c) (4 points) It was also of interest to quantitatively describe the mean sct-I concentration among cats meeting the selection criteria, prior to treatment; give a 95% confidence interval for this mean sct-I concentration. List the assumptions on which the confidence interval is based, and discuss critically whether these assumptions seem reasonable for the data at hand. If you identify concern(s) with any of the assumptions, outline how you would try to deal with the concern(s) in order to improve the analysis.
- d) (4 points) A hypothesis of clinical interest was whether the velocity values were higher (indicating a more severe condition) for cats with measurable sct-I concentrations than for cats where these concentrations were essentially zero (below the detection limit). For this question, the cats were divided into two groups based on their initial sct-I concentrations: (i) 8 cats with sct-I concentrations above zero, and (ii) 4 cats with sct-I values of zero. In the Minitab analysis for **d**), the variable defining these groups is called `sctzero`, and takes the values 0 and 1 for the above groups, respectively. Carry out a statistical analysis to examine this question, and draw conclusions. If time permits, discuss any additional analyses of interest for this question.

Minitab listing for Question 1, parts a)-c):

```
MTB > Let 'change' = 'pre'-'post'
MTB > Describe 'pre' 'post' 'change' 'velocity';
...
Descriptive Statistics: pre, post, change, velocity
```

Variable	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
pre	12	0.421	0.329	1.141	0.000	0.000	0.065	0.248	4.030
post	12	0.0375	0.0162	0.0561	0.0000	0.0000	0.0200	0.0475	0.1900
change	12	0.383	0.315	1.092	0.000	0.000	0.045	0.170	3.840
velocity	12	1.458	0.268	0.930	0.900	1.025	1.100	1.475	4.300

(continues on next page)

Minitab listing for Question 1, part d):

```
MTB > OneT 'velocity';
SUBC> Test 0;
SUBC> Confidence 95.0;
SUBC> Alternative 0.
One-Sample T: velocity
```

Test of  $\mu = 0$  vs not = 0

Variable	N	Mean	StDev	SE Mean	95% CI	T	P
velocity	12	1.458	0.930	0.268	(0.868, 2.049)	5.43	0.000

```
MTB > TwoT 'velocity' 'sctzero';
SUBC> Confidence 95.0;
SUBC> Test 0.0;
SUBC> Alternative 0.
Two-Sample T-Test and CI: velocity, sctzero
```

Two-sample T for velocity

sctzero	N	Mean	StDev	SE Mean
0	8	1.66	1.10	0.39
1	4	1.050	0.129	0.065

Difference =  $\mu(0) - \mu(1)$   
Estimate for difference: 0.612  
95% CI for difference: (-0.319, 1.544)  
T-Test of difference = 0 (vs not = 0): T-Value = 1.55 P-Value = 0.164 DF = 7

```
MTB > Mann-Whitney 95.0 'velocity_0' 'velocity_1';
SUBC> Alternative 0.
Mann-Whitney Test and CI: velocity_0, velocity_1
```

	N	Median
velocity_0	8	1.250
velocity_1	4	1.050

Point estimate for  $\eta_1 - \eta_2$  is 0.200  
96.6 Percent CI for  $\eta_1 - \eta_2$  is (-0.100, 3.100)  
W = 61.0  
Test of  $\eta_1 = \eta_2$  vs  $\eta_1$  not =  $\eta_2$  is significant at 0.1488  
The test is significant at 0.1410 (adjusted for ties)

**Question 2.** (17.5 points)

Non-response in telephone surveys (interviews) is a serious problem, both because of its potential to cause bias and because of the costs implied by unsuccessful calls. An older paper reported a study on the utility of leaving different types of messages on telephone answering machines when an initial call was answered by an answering machine. Among 1802 unsuccessful initial call attempts, 391 calls were answered by an answering machine.

- a) (5 points) Estimate the proportion of households with an (active) answering machine (at the time of the study), and supplement the estimate with a 95% confidence interval; make sure to state the statistical model/assumptions for the interval.

The authors referred to this proportion as consistent with the results and discussion in another article, in which it was stated that a value of 25% could be thought of as “roughly approximating the maximum percentage of residential telephone numbers to which an answering machine might be attached.” Do you agree with the authors’ interpretation of the observed proportion in the study? Use statistical inference to justify your answer.

In the following questions, we will focus on data collected from the 391 households where calls were picked up by an answering machine. The researchers either left one of three messages (A,B,C) or left no message. The three messages gave different levels of information about the survey, and gave notice that another call would be attempted within the next day or two. The 391 initial calls were randomly distributed onto the three message groups and a fourth group where no message was left on the machine. The table below (essentially Table 1 in the paper) gives the total number of households and the number of surveys that were eventually completed in each group, as well as totalled across the three message groups.

Message type	Number of households	Number of surveys completed
no message	100	33
message A	100	48
message B	97	43
message C	94	43
total message A+B+C	291	134

The main questions of interest were (i) whether leaving a message (of any type) would affect (in particular, improve) the response rates, and (ii) whether different levels of information in the messages would have any impact on the response rates.

- b) (4 points)

Use the data of the table and any of the attached Minitab listings to investigate question (ii). State and motivate your statistical model and hypothesis, estimate the relevant parameters, carry out a statistical test, and draw conclusions.

- c) (5 points)

While taking into account the conclusion from b), investigate question (i) in a similar manner as in b). Comment also briefly on why it might be advantageous to examine question (ii) before question (i).

In addition to the table given above, the paper also contained a Table 2, giving essentially the number of telephone interviews refused and the total number of contacts, that is, households where a potential interviewee (a person to be interviewed) could be reached.

Message type	Number of contacts	Number of refusals
no message	58	24
message A	69	21
message B	62	19
message C	69	26
total message A+B+C	200	66

The paper contains the following summary of the results (from both tables):

“The average reponse rate across all three messages (46 percent) was significantly higher ( $t=2.28$ ,  $p=.023$ ) than when no message was left (33 percent). The differences in the response rates among the three message types, though, were not statistically significant. Message households also generally had lower refusal rates (33 percent) than the no message group (41.4 percent). However, neither the individual nor the aggregate difference was statistically significant at normal ( $p < 0.05$ ) levels.”

**d)** (3.5 points)

Give a critical assessment of the summary, including both the statistical procedures referred and their interpretation. If you consider a procedure or an interpretation as incorrect, indicate also how it should be corrected.

Minitab listing for Question 2 (slightly edited):

```
MTB > XTabs;
SUBC>   Unstacked C1 - C5;
SUBC>   Counts;
SUBC>   ChiSquare;
SUBC>   Expected.
Tabulated Statistics: Worksheet rows, Worksheet columns
```

```
Rows: Worksheet rows   Columns: Worksheet columns
```

	C1	C2	C3	C4	C5	All
1	33	48	43	43	134	301
	44.13	44.13	42.81	41.49	128.43	

2	67	52	54	51	157	381
	55.87	55.87	54.19	52.51	162.57	

All	100	100	97	94	291	682
-----	-----	-----	----	----	-----	-----

Cell Contents:           Count  
                           Expected count

Pearson Chi-Square = 6.167, DF = 4, P-Value = 0.187

MTB > XTabs;

SUBC>   Unstacked C1 - C4;

...

Rows: Worksheet rows   Columns: Worksheet columns

	C1	C2	C3	C4	All
1	33	48	43	43	167
	42.71	42.71	41.43	40.15	

2	67	52	54	51	224
	57.29	57.29	55.57	53.85	

All	100	100	97	94	391
-----	-----	-----	----	----	-----

Cell Contents:           Count  
                           Expected count

Pearson Chi-Square = 5.455, DF = 3, P-Value = 0.141

MTB > XTabs;

SUBC>   Unstacked C2 - C5;

...

Rows: Worksheet rows   Columns: Worksheet columns

	C2	C3	C4	C5	All
1	48	43	43	134	268
	46.05	44.67	43.29	134.00	

2	52	54	51	157	314
	53.95	52.33	50.71	157.00	

All	100	97	94	291	582
-----	-----	----	----	-----	-----

Cell Contents:           Count  
                           Expected count

Pearson Chi-Square = 0.272, DF = 3, P-Value = 0.965

```

MTB > XTabs;
SUBC>  Unstacked C2 - C4;
...
Rows: Worksheet rows  Columns: Worksheet columns

```

	C2	C3	C4	All
1	48	43	43	134
	46.05	44.67	43.29	
2	52	54	51	157
	53.95	52.33	50.71	
All	100	97	94	291

```

Cell Contents:      Count
                   Expected count

```

Pearson Chi-Square = 0.272, DF = 2, P-Value = 0.873

```

MTB > XTabs;
SUBC>  Unstacked C1 C5;
...
Rows: Worksheet rows  Columns: Worksheet columns

```

	C1	C5	All
1	33	134	167
	42.71	124.29	
2	67	157	224
	57.29	166.71	
All	100	291	391

```

Cell Contents:      Count
                   Expected count

```

Pearson Chi-Square = 5.178, DF = 1, P-Value = 0.023

**Question 3.** (17.5 points)

In a study to describe the impact of vitamin K on blood coagulability, 10 rats received diets with varying amounts of the vitamin. The blood coagulability was measured as the (minimal) concentration of a coagulation reagent required to cause coagulation when added to the blood. The results of the study are shown in the table below, which for each rat gives the dose used and the required concentration of the coagulation reagent.

Rat no.	1	2	3	4	5	6	7	8	9	10
Vitamin K dose ( $x$ )	1.51	2.14	2.63	3.37	4.47	5.62	6.76	8.32	10.23	10.96
Reagent conc. ( $y$ )	446.7	177.8	182.0	89.1	52.5	39.8	35.5	20.9	13.5	11.7

Different types of relationships may be hypothesized between the vitamin K dose ( $x$ ) and the concentration of the coagulation reagent ( $y$ ); the list below describes four such relationships, where  $\times$  denotes multiplication, and  $\log(y)$  denotes the natural logarithm of  $y$ .

- 1)  $y = a + b \times x$ ,
- 2)  $y = a \times x^b$ ,                    or equivalently:  $\log(y) = \log(a) + b \times \log(x)$ ,
- 3)  $y = a \times b^x$ ,                    or equivalently:  $\log(y) = \log(a) + \log(b) \times x$ ,
- 4)  $y = a/[1 + (b \times x)]$ ,        or equivalently:  $1/y = 1/a + (b/a) \times x$ .

**a)** (5.5 points)

Review the five analyses in the Minitab listings and explain the statistical models they are based on. (*Hint:* You don't need to repeat the details of the models for all analyses, as long as you explain clearly how the models differ.) Briefly discuss how well you think each of these models are suited for the data, and select the model/analysis you think gives the best description of the data. Make sure to motivate your choice, for example by explaining why you consider the other models as inferior to your chosen model.

**b)** (5 points)

Estimate the parameters of your selected ("best") model from **a)**, and supplement the estimates with 90% confidence intervals, where possible. Additionally, compute estimates and 90% confidence intervals for the parameters  $a$  and  $b$  of the relation among 2)-4) above corresponding to your model; if your preferred model does not correspond to any of the relations 2)-4), choose for this step instead your second best model from **a)**.

**c)** (3 points)

Based on your selected ("best") model from **a)**, give a statistical assessment of whether the data provide evidence of the expected (well-known) relation between vitamin K and blood coagulability, whereby deficiency in vitamin K seriously impacts blood coagulation and higher levels of vitamin K facilitate blood coagulation. State your statistical hypotheses and procedures carefully, and draw conclusions.

**d)** (4 points)

In another experiment, the value 27.5 for the concentration of the coagulation reagent was recorded for a rat that had received a vitamin K dose of 6.17. Examine, based on your selected ("best") model from **a)**, whether this observation seems to be in agreement with the current data.

Minitab listings for Question 3 (condensed and rearranged to save space):

```
MTB > Let 'lndose' = ln('dose')
MTB > Let 'lnreagent' = ln('reagent')
MTB > Let 'invreagent' = 1/'reagent'
MTB > Print 'dose'-'invreagent'.
```

Data Display

Row	dose	reagent	lndose	lnreagent	invreagent
1	1.51	446.7	0.41211	6.10189	0.0022386
2	2.14	177.8	0.76081	5.18066	0.0056243
3	2.63	182.0	0.96698	5.20401	0.0054945
4	3.47	89.1	1.24415	4.48976	0.0112233
5	4.47	52.5	1.49739	3.96081	0.0190476
6	5.62	39.8	1.72633	3.68387	0.0251256
7	6.76	35.5	1.91102	3.56953	0.0281690
8	8.32	20.9	2.11866	3.03975	0.0478469
9	10.23	13.5	2.32532	2.60269	0.0740741
10	10.96	11.7	2.39425	2.45959	0.0854701

```
MTB > Fitline 'reagent' 'dose'.
Regression Analysis: reagent versus dose
```

The regression equation is  
 $\text{reagent} = 272.7 - 29.53 \text{ dose}$

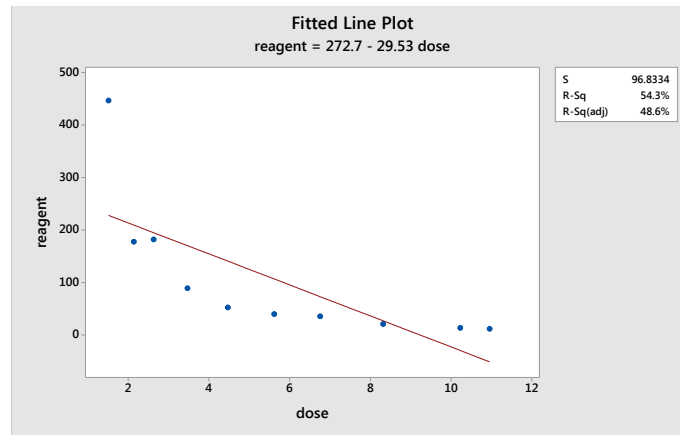
S = 96.8334    R-Sq = 54.3%    R-Sq(adj) = 48.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	89176	89176.4	9.51	0.015
Error	8	75014	9376.7		
Total	9	164190			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	272.7	61.8	4.41	0.002
dose	-29.53	9.58	-3.08	0.015



```
MTB > Predict 'reagent';
```

```
...
Variable Setting      Fit   SE Fit      95% CI      95% PI
dose      6.17    90.4411  31.0858  (18.7571, 162.125)  (-144.081, 324.963)
dose      3.31   174.905  37.7258  (87.9094, 261.901)  (-64.7411, 414.551)
dose      1.82   218.909  47.4940  (109.388, 328.431)  (-29.8016, 467.620)
```

(continues on next two pages)

MTB > Fitline 'lnreagent' 'dose'.  
 Regression Analysis: lnreagent versus dose

The regression equation is  
 $\text{lnreagent} = 5.956 - 0.3433 \text{ dose}$

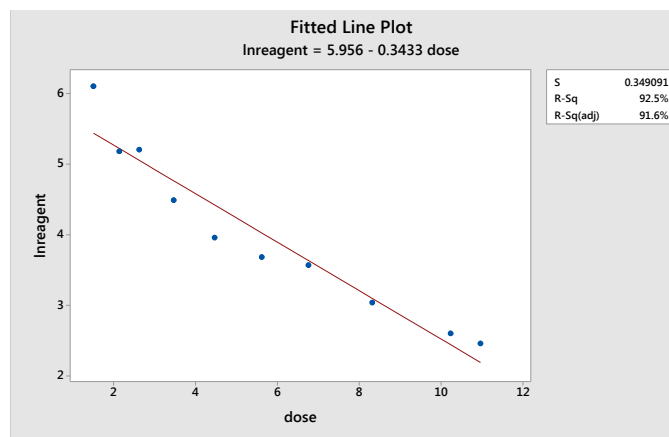
S = 0.349091 R-Sq = 92.5% R-Sq(adj) = 91.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	12.0521	12.0521	98.90	0.000
Error	8	0.9749	0.1219		
Total	9	13.0270			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	5.956	0.223	26.71	0.000
dose	-0.3433	0.0345	-9.94	0.000



MTB > Predict 'lnreagent';

...

Variable	Setting	Fit	SE Fit	95% CI	95% PI
dose	6.17	3.83733	0.112066	(3.57891, 4.09576)	(2.99187, 4.68280)
dose	3.31	4.81926	0.136004	(4.50563, 5.13288)	(3.95532, 5.68320)
dose	1.82	5.33082	0.171219	(4.93599, 5.72565)	(4.43420, 6.22744)

MTB > Fitline 'lndose' 'lnreagent'.  
 Regression Analysis: lndose versus lnreagent

The regression equation is  
 $\text{lndose} = 3.797 - 0.5611 \text{ lnreagent}$

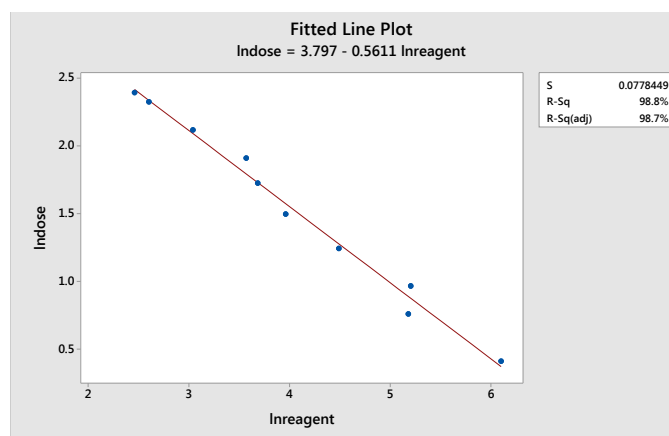
S = 0.0778449 R-Sq = 98.8% R-Sq(adj) = 98.7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	4.10201	4.10201	676.92	0.000
Error	8	0.04848	0.00606		
Total	9	4.15049			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	3.7967	0.0903	42.04	0.000
lnreagent	-0.5611	0.0216	-26.02	0.000



MTB > Predict 'lndose';

...

Variable	Setting	Fit	SE Fit	95% CI	95% PI
lnreagent	6.17	0.334433	0.0523238	(0.213774, 0.455092)	(0.118140, 0.550726)
lnreagent	3.31	1.93931	0.0290969	(1.87221, 2.00641)	(1.74767, 2.13095)
lnreagent	1.82	2.77542	0.0536322	(2.65174, 2.89909)	(2.55743, 2.99341)

MTB > Fitline 'lnreagent' 'ldose'.  
 Regression Analysis: lnreagent versus ldose

The regression equation is  
 $\text{lnreagent} = 6.734 - 1.761 \text{ ldose}$

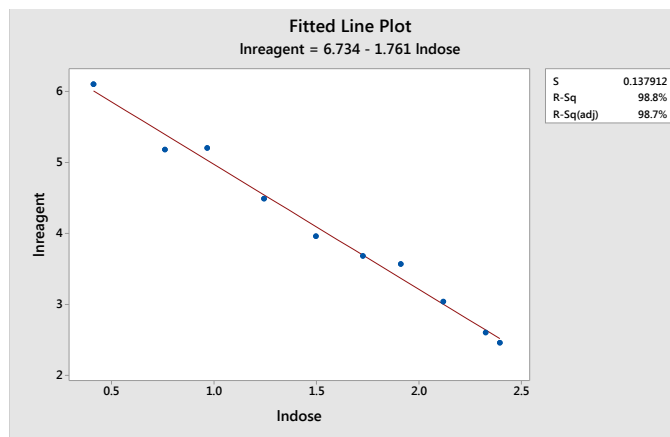
S = 0.137912 R-Sq = 98.8% R-Sq(adj) = 98.7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	12.8749	12.8749	676.92	0.000
Error	8	0.1522	0.0190		
Total	9	13.0270			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	6.734	0.113	59.73	0.000
ldose	-1.7613	0.0677	-26.02	0.000



MTB > Predict 'lnreagent';

Variable	Setting	Fit	SE Fit	95% CI	95% PI
ldose	6.17	-4.13291	0.316733	(-4.86330, -3.40252)	(-4.92953, -3.33629)
ldose	3.31	0.904272	0.127783	(0.609604, 1.19894)	(0.470717, 1.33783)
ldose	1.82	3.52854	0.0476693	(3.41861, 3.63846)	(3.19205, 3.86503)

MTB > Fitline 'invreagent' 'dose'.  
 Regression Analysis: invreagent versus dose

The regression equation is  
 $\text{invreagent} = -0.01735 + 0.008516 \text{ dose}$

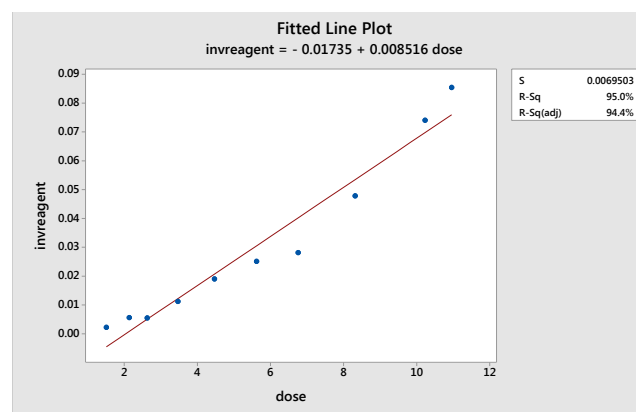
S = 0.00695028 R-Sq = 95.0% R-Sq(adj) = 94.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	0.0074147	0.0074147	153.49	0.000
Error	8	0.0003865	0.0000483		
Total	9	0.0078011			

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	-0.01735	0.00444	-3.91	0.004
dose	0.008516	0.000687	12.39	0.000



MTB > Predict 'invreagent';

Variable	Setting	Fit	SE Fit	95% CI	95% PI
dose	6.17	0.0351918	0.0022312	(0.0300466, 0.0403369)	(0.0183588, 0.0520247)
dose	3.31	0.0108365	0.0027078	(0.0045923, 0.0170807)	(-0.0063643, 0.0280373)
dose	1.82	-0.0018521	0.0034089	(-0.0097131, 0.0060089)	(-0.0197035, 0.0159993)