Supplementary exercise 1.110 of IPS7e

We first demonstrate the calculations when using a statistical table for the standard normal distribution, e.g. Table B of PSLS. We denote by Z a random variable from $\mathrm{N}(0,1)$, the standard normal distribution.

(a) $P(Z \leq -2) = 0.0228$,

(b) $P(Z \geq -2) = 1 - P(Z \leq -2) = 1 - 0.0228 = 0.9772$, (or directly as $P(Z \leq 2)$),

(c) $P(Z > 1.67) = 1 - P(Z < 1.67) = 1 - 0.9525 = 0.0475$, (or directly as $P(Z < -1.67)$),

(d) $P(-2 < Z < 1.67) = P(Z < 1.67) - P(Z \leq -2) = 0.9525 - 0.0228 = 0.9297$.

Note that for all these calculations, it does not matter whether the endpoints of the intervals are included or not; this is because the probability of any single value (such as the interval endpoints) is zero in a continuous probability distribution (such as the normal distribution).

Next we demonstrate two ways in which Minitab can be used for the calculations. By the first method, we essentially replace the lookup in a table for cumulative probabilities (i.e., probabilities "to the left" of the form $P(Z \leq -2)$) by calculation by a Minitab function. The function is found in the `Calc-Probability Distributions` menu, after choosing the normal distribution and cumulative probability and entering the desired value for which we want to compute the cumulative probability as "Input constant". Note that only cumulative probabilities can be computed by this function (just as in the table), and probabilities for other sets need to be computed by combining these as shown in the calculations above. Minitab commands and output for the first calculation from above:

```
MTB > CDF -2;
SUBC>   Normal 0.0 1.0.
```

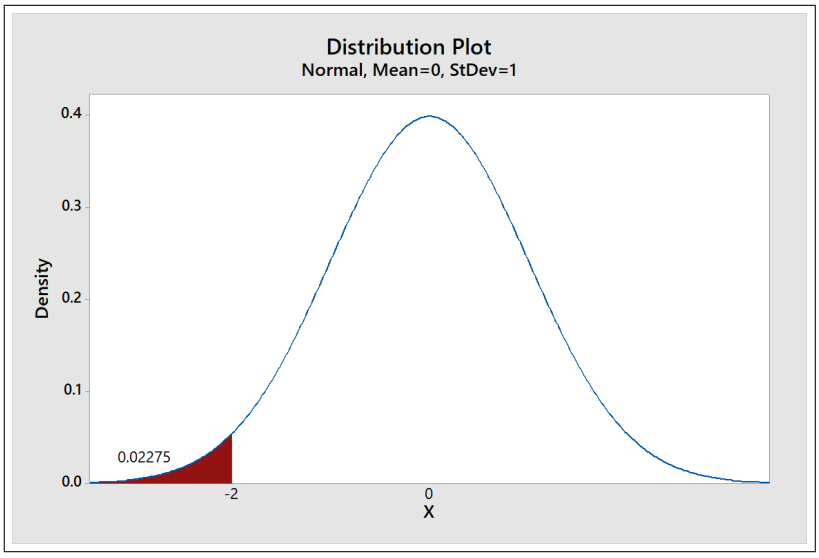
Cumulative Distribution Function

Normal with mean = 0 and standard deviation = 1

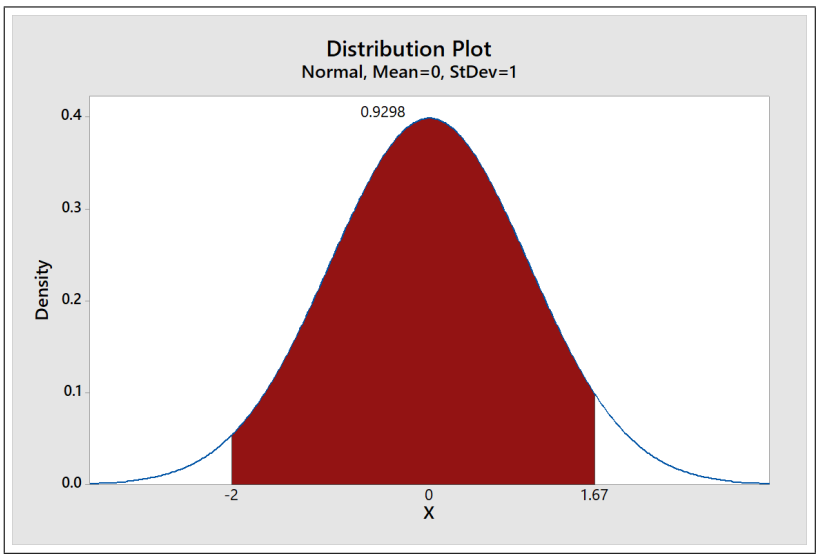
x	P(X ≤ x)
-2	0.0227501

The second way to calculate probabilities in Minitab uses the graphical display in the menu obtained from `Graph-Probability Distribution Plot`. Select the plot type by "View Probability" and "Shared Area", after having selected the probability distribution of interest. The default is to determine the shaded area by the probability, corresponding to an inverse probability calculation where one determines the value (percentile) of a distribution corresponding to a certain probability. Probability calculations are obtained after changing to "X value", and the type of area for the calculation can be selected. The Minitab commands for these plots are long and technical, and not particularly helpful, so only the graphical outputs are shown here (on the next page), corresponding to parts (a) and (d).

Part (a)


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
0.02275
-2
0
X

Part (d)


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
0.9298
-2
0
1.67
X