Supplementary exercises 12.54 and 12.55 of IPS7e

Data: Measurements of vitamin contents in loaves immediately after baking and after storage for 1, 3, 5 and 7 days. Two loaves were measured for each storage condition. In this exercise we consider two outcomes: concentrations of vitamin A and vitamin C.

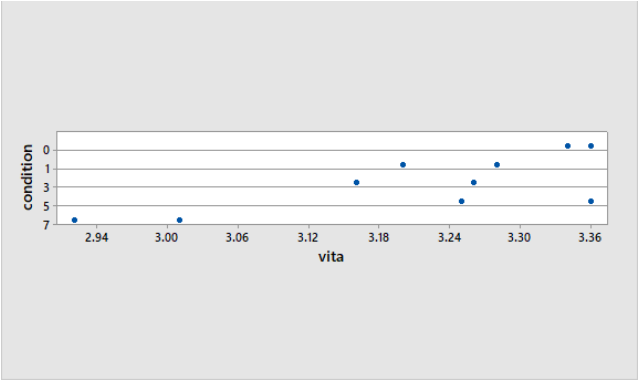
Model: If we denote the vitamin concentration in loaf j subjected to storage condition i (where $i = 1, 2, 3, 4, 5$ and $j = 1, 2$), the statistical model is that the X_{ij}'s are normally distributed $\mathrm{N}(\mu_i, \sigma)$, and all observations are independent.

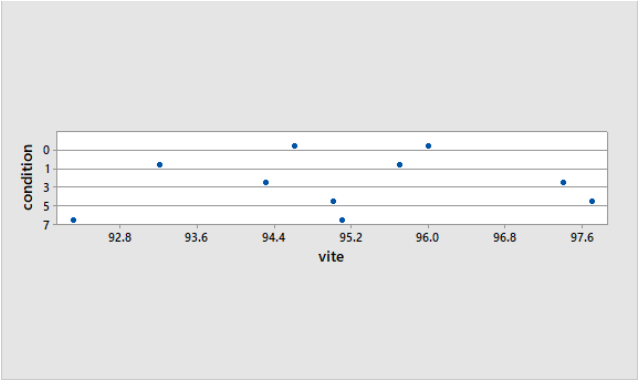
With only two replicates per group, it is almost impossible to check the assumptions of the model. The standard deviations within groups are highly variable when based only on two observations, and the IPS/PSLS guideline for the homogeneity of standard deviations can be dispensed with. The best descriptive statistic is a dotplot to get a visual impression of whether the groups looks reasonable similar. We give Minitab commands and outputs for the descriptive analysis.

```
MTB > Describe 'vita' 'vite';
SUBC>   By 'condition'.
MTB > Dotplot ( 'vita' 'vite' ) * 'condition'.
```

Statistics

Variable	condition	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
vita	0	2	0	3.3500	0.0100	0.0141	3.3400	*	3.3500	*	3.3600
	1	2	0	3.2400	0.0400	0.0566	3.2000	*	3.2400	*	3.2800
	3	2	0	3.2100	0.0500	0.0707	3.1600	*	3.2100	*	3.2600
	5	2	0	3.3050	0.0550	0.0778	3.2500	*	3.3050	*	3.3600
	7	2	0	2.9650	0.0450	0.0636	2.9200	*	2.9650	*	3.0100
vite	0	2	0	95.300	0.700	0.990	94.600	*	95.300	*	96.000
	1	2	0	94.45	1.25	1.77	93.20	*	94.45	*	95.70
	3	2	0	95.85	1.55	2.19	94.30	*	95.85	*	97.40
	5	2	0	96.35	1.35	1.91	95.00	*	96.35	*	97.70
	7	2	0	93.70	1.40	1.98	92.30	*	93.70	*	95.10





Comments:
The variation is reasonably constant across the groups. Except for the 7-day storage group and the vitamin A outcome, the groups also seem to have approximately the same location.

Exercise 12.54
We proceed with the analysis of variance, focusing at first on the ANOVA table and the group means. The null and alternative hypotheses are:

$$\begin{aligned} \mathrm{H}_0: &\quad \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 \quad \text{(same mean vitamin level in all storage groups)}, \\ \mathrm{H}_a: &\quad \text{not } \mathrm{H}_0, \text{ that is, some storage groups differ.} \end{aligned}$$

One-way ANOVA: vita versus condition

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
condition	5	0, 1, 3, 5, 7

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
condition	4	0.17894	0.044735	12.09	0.009
Error	5	0.01850	0.003700		
Total	9	0.19744			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0608276	90.63%	83.13%	62.52%

Means

condition	N	Mean	StDev	95% CI
0	2	3.3500	0.0141	(3.2394, 3.4606)
1	2	3.2400	0.0566	(3.1294, 3.3506)
3	2	3.2100	0.0707	(3.0994, 3.3206)
5	2	3.3050	0.0778	(3.1944, 3.4156)
7	2	2.9650	0.0636	(2.8544, 3.0756)

Pooled StDev = 0.0608276

Interval Plot of vita vs condition

One-way ANOVA: vite versus condition

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
condition	5	0, 1, 3, 5, 7

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
condition	4	9.086	2.272	0.69	0.630
Error	5	16.475	3.295		
Total	9	25.561			

Model Summary

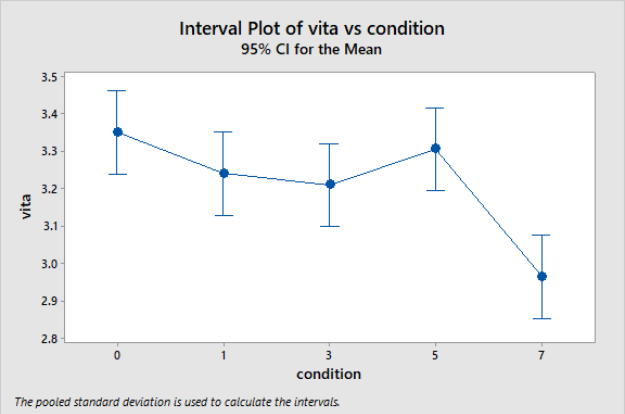
S	R-sq	R-sq(adj)	R-sq(pred)
1.81521	35.55%	0.00%	0.00%

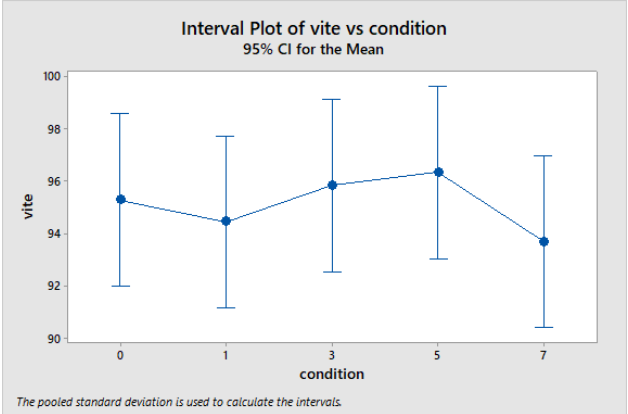
Means

condition	N	Mean	StDev	95% CI
0	2	95.300	0.990	(92.001, 98.599)
1	2	94.45	1.77	(91.15, 97.75)
3	2	95.85	2.19	(92.55, 99.15)
5	2	96.35	1.91	(93.05, 99.65)
7	2	93.70	1.98	(90.40, 97.00)

Pooled StDev = 1.81521

Interval Plot of vite vs condition





Comments:
The ANOVA tables show that the F-tests have (4,5) degrees of freedom, and have values of 12.09 and 0.69 for vitamin A and E, respectively. The former is clearly significant, and the latter is clearly nonsignificant.

For vitamin A we conclude that some differences among groups exist, and looking at the means it does in fact seem as if the bread has lost some of its vitamin A after a 7-day storage (but not after 5 days). For vitamin E we conclude that there is no evidence of a difference among the storage groups; the bread does not seem to loose its vitamin E by storage (up to 7 days).

Exercise 12.55

(a) It is inappropriate to perform multiple comparisons when the overall F-test is nonsignificant. When there is no (overall) evidence of a difference between the groups (and no apriori hypotheses), it makes no sense to go "hunting" after specific differences between groups.

(b) Assuming that no particular group differences are of special interest, we may use the Bonferroni method to account for a total of $5 \cdot 4/2 = 10$ pairwise comparisons. Therefore, the individual error rate is $0.05/10 = 0.005$.

An alternative to using Minitab is to compute an adjusted LSD value directly. From the t-distribution table with df $= 5$ (DFE) and confidence level $C = 99.5\%$ (corresponding to an error level of 0.5%) we get $t^* = 4.773$. Therefore, the LSD-value adjusted for multiple comparisons equals

$$\text{LSD}(.995) = 4.773 \cdot 0.06083\sqrt{2/2} = 0.29.$$

Among the group means, only those of group 7 and 0 days, and of group 7 and 5 days, differ by more than 0.29. Therefore these are the only significant comparisons. Further comments below the Minitab print, showing only the extra listing resulting from the Comparisons submenu, where the Fisher error rate has been set to 0.005 (not 0.5, as one would intuitively think).

Fisher Pairwise Comparisons

Grouping Information Using the Fisher LSD Method and 99.5% Confidence

condition	N	Mean	Grouping	
0	2	3.3500	A	
5	2	3.3050	A	
1	2	3.2400	A	B
3	2	3.2100	A	B
7	2	2.9650		B

Means that do not share a letter are significantly different.

Fisher Individual Tests for Differences of Means

Difference of Levels	Difference of Means	SE of Difference	99.5% CI	T-Value	Adjusted P-Value
1 - 0	-0.1100	0.0608	(-0.4004, 0.1804)	-1.81	0.130
3 - 0	-0.1400	0.0608	(-0.4304, 0.1504)	-2.30	0.070
5 - 0	-0.0450	0.0608	(-0.3354, 0.2454)	-0.74	0.493
7 - 0	-0.3850	0.0608	(-0.6754, -0.0946)	-6.33	0.001
3 - 1	-0.0300	0.0608	(-0.3204, 0.2604)	-0.49	0.643
5 - 1	0.0650	0.0608	(-0.2254, 0.3554)	1.07	0.334
7 - 1	-0.2750	0.0608	(-0.5654, 0.0154)	-4.52	0.006
5 - 3	0.0950	0.0608	(-0.1954, 0.3854)	1.56	0.179
7 - 3	-0.2450	0.0608	(-0.5354, 0.0454)	-4.03	0.010
7 - 5	-0.3400	0.0608	(-0.6304, -0.0496)	-5.59	0.003

Simultaneous confidence level = 97.46%

Interval Plot of vita vs condition

Comments:
The 99.5% confidence intervals lead to the same conclusion as described above, namely that the

only significant comparisons are 7-0 and 7-5 (these are the only CIs not including zero). The grouping information represents that conclusion in a diagram. The column labelled "Adjusted P-value" is a bit misleading here, because the Fisher method involves no adjustment. Therefore the P-values are unadjusted for multiple testing, and our choice of a 0.5% error level has no impact on these P-values. The correct way to interpret these is therefore at a 0.5% significance level, showing us that only the two comparisons 7-0 and 7-5 are significant.

Our results are that at the overall 5% level there is a significant difference between 7-day storage and storage of 0 and 5 days, but not 1 and 3 days. Biologically this does not make much sense, so a more reasonable conclusion is that the data show a lower vitamin A content for day 7 than all other days, and this effect is significant at the overall 5% level for 0 and 5 days, and close to significant for days 1 and 3. The Bonferroni method is conservative, so other methods might give significance for more days (for example, the Tukey method shows significance for all comparisons with 7 days), but in practice this should be of little importance for the conclusions from the study.