

Solution to home assignment II

The actual study focused on the potential performance enhancement by scents. The subjects were equipped with masks, which could either have a floral scent or be unscented. For this solution, we will refer directly to the actual meaning of the two groups. The subjects worked on completing pencil and paper mazes with the two types of masks.

a. Study design

The study is an experiment in which the subjects are assigned to the two treatments (scented and unscented masks). The study design is a cross-over trial, and could also be described as a block design with the subjects as blocks. The multiple observations obtained from the same subject are “paired” and definitely not independent, neither within nor between the two trial periods.

Randomization would apply to the order in which subjects performed tasks, i.e. whether they started with the scented or unscented mask. Half of the subjects started with the scented mask, and for a proper randomization they should have been randomly selected among all subjects. Blinding on the other hand seems difficult to achieve because the subjects and experimentors would naturally know which mask was worn at any given time.

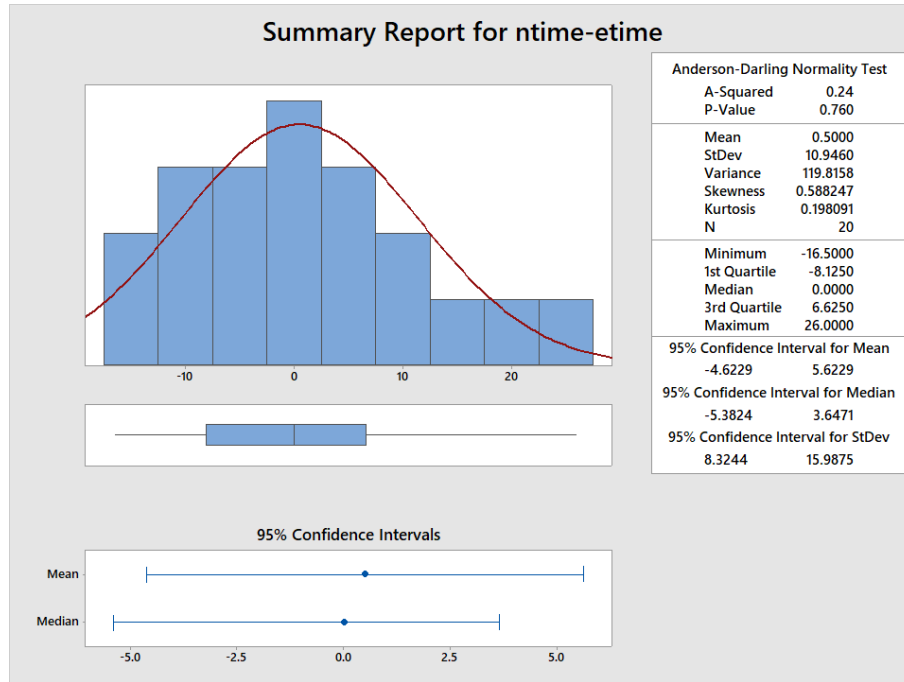
If the order in which trials were performed was important, it could reduce potential confounding if the two possible orders were reasonably balanced with respect to subject characteristics that could affect performance. The data include information on sex and age. Six women and four men started with the scented mask, whereas four women and six men started with the unscented mask; while not perfectly balanced, this is still quite close. Also the age distributions for the two order groups seem quite similar, with the mean age 4.3 years higher in the EN group and a somewhat lower spread than in the NE group ($s = 11.0$ and 14.3 for EN and NE, respectively). In summary, both demographic variables seem “reasonably balanced” onto the order groups (no statistical inference was expected for this part).

b. Effect of scent on completion times

The desired variable is the difference between the average completion times in the two scented (enhanced) trials and the two unscented (neutral) trials. For example, for the first subject the value is

$$D_1 = (76 + 48)/2 - (58 + 58)/2 = 4.0.$$

A descriptive analysis is based on the figure on the next page. The distribution looks unimodal and mildly right-skewed, and centered around zero (or a value close to zero). Half the subjects are faster with the scented masks and half are faster with the unscented masks (because the median equals zero). On the average, there is hardly any difference in completion times with the scented and unscented masks (because the center is close to zero). From the plot and these descriptive statistics it does not appear that the scent helps the subjects to work the maze faster.



Denote these differences by D_1, \dots, D_{20} . We assume the differences to constitute a simple random sample from a normal distribution $N(\mu, \sigma)$ with unknown mean μ and standard deviation σ . Despite the right-skewness, the normality test gives no evidence against a normal distribution, and we should be fine to use inference based on the normal distribution. The hypothesis of interest is $H_0 : \mu = 0$, and we take a two-sided alternative hypothesis $H_a : \mu \neq 0$. Even though the researchers believe the enhancement could lead to faster completion times, it is perfectly possible that the opposite could happen. To assess the hypothesis we either compute a 95% CI and check whether zero is inside, or compute a t -test:

$$95\% \text{ CI: } \bar{D} \pm t^* s_D / \sqrt{n} = 0.50 \pm 5.12 = (-4.62, 5.62),$$

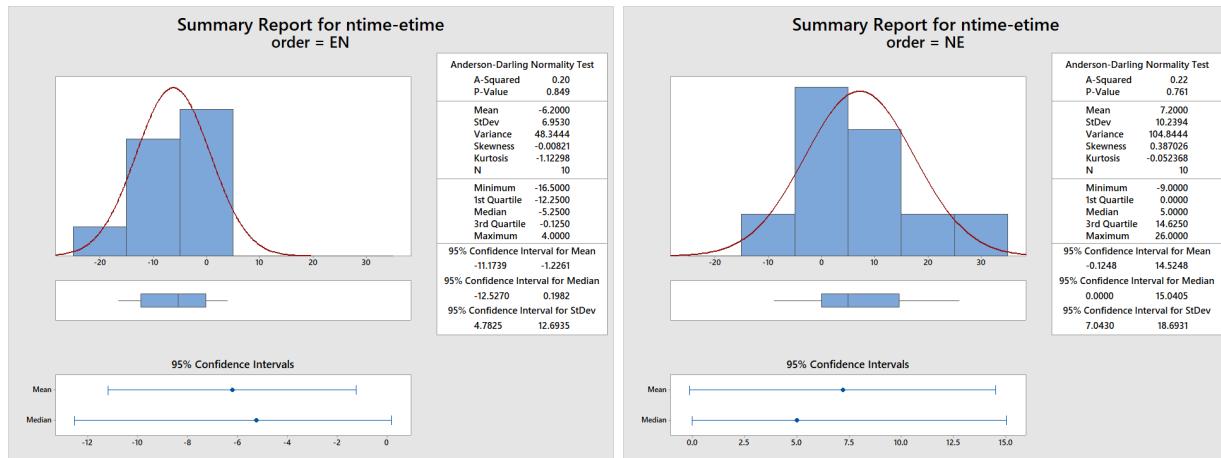
$$t = \frac{\bar{D} - 0}{s_D / \sqrt{n}} = 0.20, \quad P = 2 \cdot P(t(19) > 0.20) = 0.84.$$

As zero is clearly inside the confidence interval and/or the P -value of the test is clearly non-significant, this analysis gives no indication, and certainly no evidence, that the subjects complete the work at different speeds within the two environments.

c. Effects of scent and order on completion times

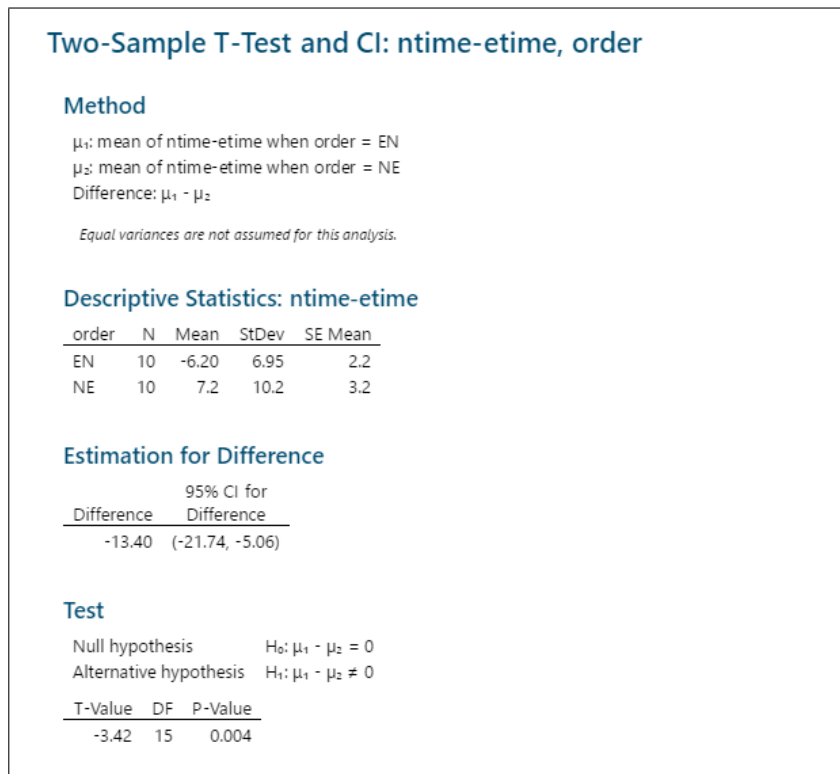
The 20 differences considered in Question b) are not as homogeneous as it might seem at first glance. Ten of the subjects did the unscented trials first, and the ten remaining subject did the scented trials first. If there was a learning effect it would be more appropriate to consider these as two independent samples (of differences). The samples are independent because they are for different subjects. To illustrate the difference between the two samples we use the same summary for each group. Other

useful displays are dotplots and boxplots for both groups.



The two distributions both look fairly symmetric and bell-shaped, although the distribution shape is difficult to assess from so small samples. It is clear though that the distributions are centered around different values. If the scented trials were done first (order = EN) the scented trials took around 6 units (seconds?) longer than the scented trials, but in the opposite order the unscented trials took around 7 units longer.

The statistical analysis should now be based on assuming two independent samples, and as the distributions show no apparent deviations from normal distributions (and clearly non-significant P -values of the normality tests) we also assume normal distributions $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$, respectively. There is no real need to assume $\sigma_1 = \sigma_2$. The hypothesis of interest is $H_0 : \mu_1 = \mu_2$ (corresponding to no effect of the order), and we take again a two-sided alternative $H_a : \mu_1 \neq \mu_2$.



There is a very clear and highly significant impact of the order. The confidence interval for the

difference of the means is far from zero, and the P -value is very low. There does indeed seem to be a learning effect because the mask used first has the average highest completion times. This finding also implies that the analysis of Question b) is inadequate (because the model assumption that subjects within each scent group were identically distributed was not met). We will adjust the comparison between scented and unscented trials for the order effect in Question d.i).

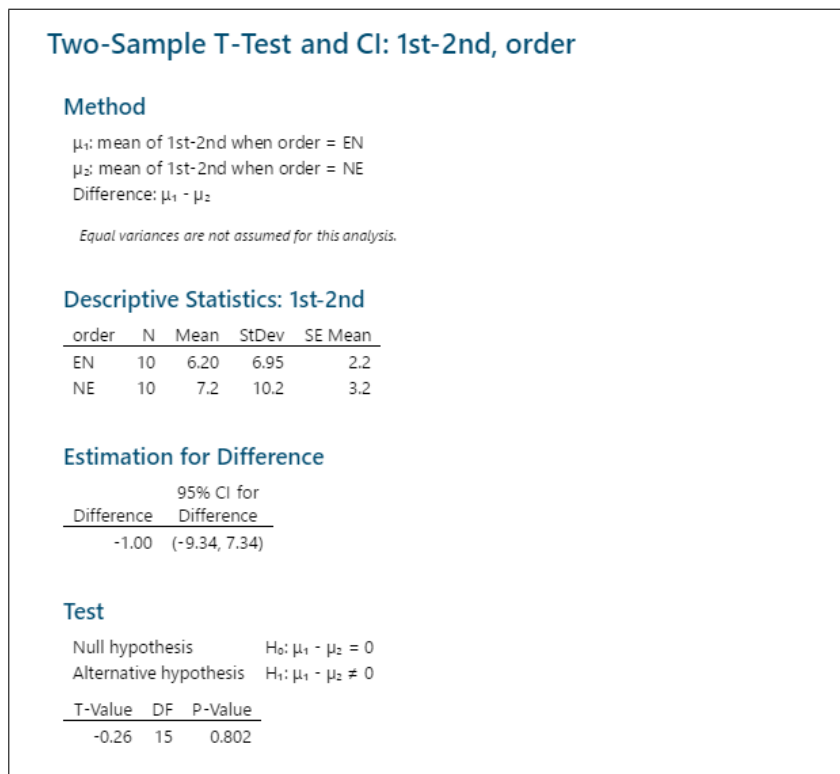
d.i. Further analyses on the effects of scent and order on completion times

In presence of the strong order effect, it might be more useful to measure the difference between first and second sets (each consisting of two trials with one of the two masks). We therefore compute the difference between the average completion times in the first two trials and the last two trials, and denote these by E_i . For example, the first subject did the scented (enhanced) trials first so the value equals

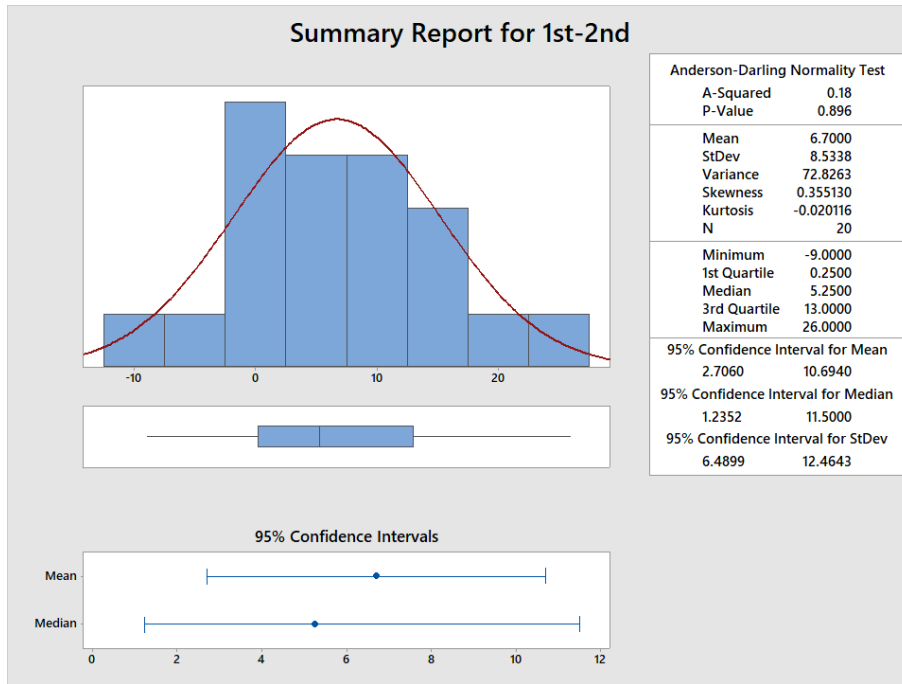
$$E_1 = (58 + 58)/2 - (76 + 48)/2 = -4.0,$$

in other words *minus* the previously computed difference. For the subjects who did the unscented trials first, the value is the *same* as the difference previously computed. In formulae, we have $E_i = D_i$ for order NE, and $E_i = -D_i$ for order EN.

If there also was a real impact of the masks, it would show up as an order effect. For example, if completion times were generally faster with the scented mask, the difference between first and second trials would be larger for subjects who used the unscented mask first than for subjects who used the scented mask first. If completion times were slower with scented masks, the largest differences would be seen when scented masks were worn first. Our interest is therefore in comparing the two orders. As the distributions for the two orders are unchanged and sign-reversed, respectively, compared to the previously shown distributions, we skip the descriptive statistics and proceed directly to the test. We again assume normal distributions, and test equality between the two orders against a two-sided alternative, using a two-sample t -test.



The confidence interval for the difference between the means is very wide and easily includes zero, and the test statistic is clearly non-significant. There is no real indication of a difference between completion times with the two masks. As a follow-up on this analysis, we show descriptive statistics for the differences between the first and second trials.



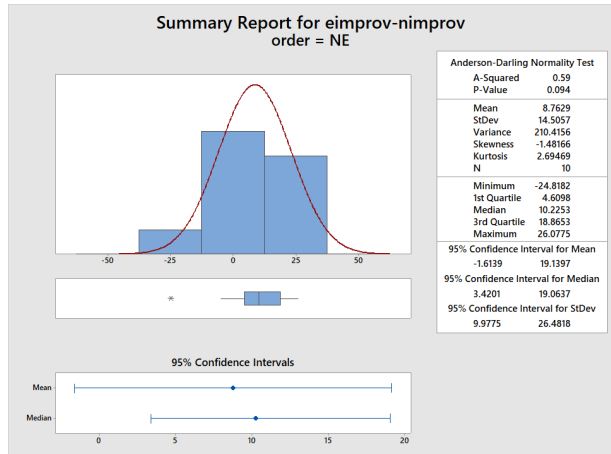
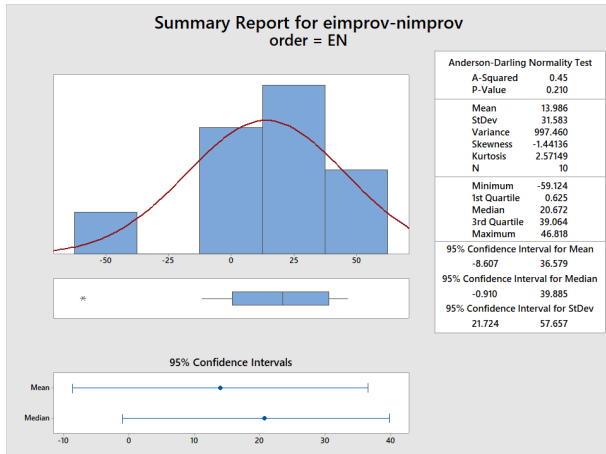
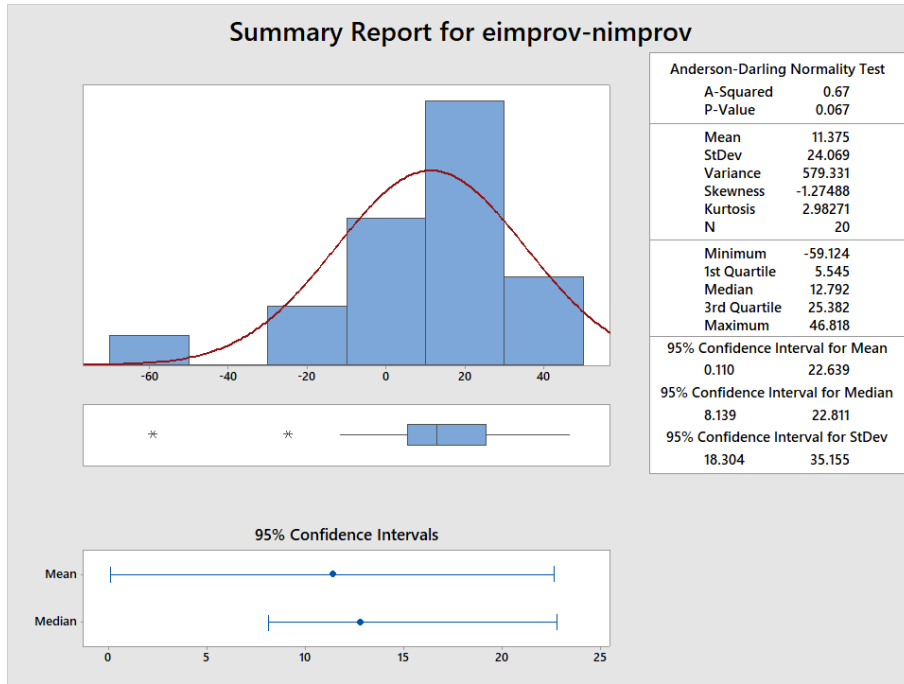
The figure shows that the majority of subjects completed the work faster in the second trial, on the average 6.7 units faster. The distribution looks fairly normal so we may consider the 95% confidence interval for the mean of (2.7,10.7) as exact. There is evidence of this being a real effect from the CI (we could also compute a one-sample t -test for a mean of zero). To reiterate our conclusions, we could say that subjects on the average complete the second trial faster than the first, but that there is no indication in these data of a difference between average completion times with scented and unscented masks.

d.ii. Effects of scent and order on speed of learning

Within one set of trials, we may compute the improvement in speed as the difference between the last and first completion times, and then divide by the first completion time and multiply by 100 to express the improvement in percent. For example, the first subject has the following improvements in unscented and scented trials,

$$\text{scented: } \frac{76 - 48}{76} \times 100\% = 36.8\%, \quad \text{unscented: } \frac{58 - 58}{58} \times 100\% = 0\%.$$

In a similar fashion as above we could now consider the differences between these rates, either as “unscented – scented” or as “first – second”. Given the earlier results one would probably choose the last form, or both, but following the way the question was phrased we consider the former. The summaries below describe the distribution, both overall and separated into order groups.



Our first impression from these figures is that both distributions appear centred at values above zero, but they also each include one extreme observation in the left tail with a large negative value (subjects 4 and 16). As it is not obvious that inference based on normal distributions will be valid (the two-sample $t = 0.48$ with $df = 12$ is clearly non-significant), one may instead try the nonparametric WMW-test ($P = 1$). Both analyses therefore offer absolutely no evidence of an impact of order. Also for the combined sample the issue around assuming normality remain, and it may seem more natural to focus on the median for the difference in the rate of improvement between scented and unscented trials. The estimated median is 12.8, corresponding to a 12.8% larger improvement in the scented trials, and its 95% confidence interval of (8.1, 22.8) is well away from zero. Additionally, it could be suggested to further explore the two extreme observations of slower learning in the scented trials.

We conclude that there seems to be evidence of faster learning with scented masks than with unscented masks, whereas the order of the trials seems to be of no importance for the learning rates.