# Index of 10-L

(slides 12/13 swapped, some edits on (new) slide 13)

# Practical Information

Major news:

- midterm: done... — do we need a review?
- home assignment II: to be returned to you on Monday,
- home assignment III has been posted: worth 10% of course mark, deadline Thursday 7/11,
- optional project proposal: also due Thursday 7/11,
- now jumping ahead in textbooks:
  we'll come back to regression and correlation.

Today's lecture — another new method/analysis...

- main topic: 1-way ANOVA (analysis of variance),[1]
- contrasts are not part of the course curriculum![2]
- jump over: references to residuals and $R^2$,
  (we'll talk about these later)
- additional topic: Kruskal-Wallis test (PSLS Chapter 27) — non-parametric 1-way ANOVA,
- new: "misconceptions clinic" (discussion based on Greenland et al.'s 2016 paper; media page).

[1] PSLS 4e: Chapters 24 & 26; S: Section 11.3 (too briefly); IPS 7e: Chapter 12.
[2] Contrasts are useful!: Yossa & Verdegem (2015), *Aquaculture* **437**, 344−350.

# Data Example(s): Reading Scores

Teaching reading comprehension[3]:

- 66 students, randomly assigned to one of 3 teaching groups, with 22 students in each group,
  - **B**asal: traditional method,
  - **D**RTA and **S**trat: innovative methods,
- 2 pre-tests (before teaching) and 3 post-tests (after teaching),
- questions of interest: compare the 3 groups, in particular the new ones versus **B**asal, and also **D**RTA versus **S**trat,
- in the lecture, we look at `pre1` and `post3`.

Outline of data set:

| variable | group | observations (22 in each row) | | | | | | | mean | sd |
|---|---|---|---|---|---|---|---|---|---|---|
| **B**asal | `pre1` | 4 | 6 | 9 | 12 | 16 | ... | 9 | 10.5 | 2.97 |
| | `post3` | 41 | 41 | 43 | 46 | 46 | ... | 32 | 41.0 | 5.64 |
| **D**RTA | `pre1` | 7 | 7 | 12 | 10 | 16 | ... | 10 | 9.7 | 2.69 |
| | `post3` | 31 | 40 | 48 | 30 | 42 | ... | 49 | 46.7 | 7.39 |
| **S**trat | `pre1` | 11 | 7 | 4 | 7 | 7 | ... | 8 | 9.1 | 3.34 |
| | `post3` | 53 | 47 | 41 | 49 | 43 | ... | 42 | 44.3 | 5.77 |

[3] IPS textbook dataset, from a study conducted by Jim Baumann and Leah Jones at the Purdue University School of Education.

# Notation for 1-Way ANOVA

Data layout and notation:

| group | | observations ($j$) | | | | | number | mean | std.dev. |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | $X_{11}$ | $X_{12}$ | $\ldots$ | $X_{1n_1}$ | | $n_1$ | $\bar{X}_1$ | $s_1$ |
| row | 2 | $X_{21}$ | $X_{22}$ | $\ldots$ | $\ldots$ | $X_{2n_2}$ | $n_2$ | $\bar{X}_2$ | $s_2$ |
| ($i$) | $\vdots$ | $\vdots$ | $\vdots$ | $\ldots$ | $\ldots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| | $I$ | $X_{I1}$ | $X_{I2}$ | $\ldots$ | $X_{In_I}$ | | $n_I$ | $\bar{X}_I$ | $s_I$ |

- $X_{ij} = j$th observation in $i$th group (row), where
  - $i = 1, \ldots, I$, and $I =$ number of groups (rows),
  - $j = 1, \ldots, n_I$, and $n_i =$ number of obs. in $i$th group.
- denote also by $N = n_1 + \ldots + n_I$ the total number of observations, and by $\bar{X} = \sum_{ij} X_{ij}/N$ the overall mean,
- the dataset/design is
  - balanced, if all groups equally large, ($n_1 = \ldots = n_I$),
  - unbalanced, otherwise (some groups of different size),

  balanced designs are *nice* and simpler, but when using a computer unbalancedness is not a problem.

The natural model would seem to be (assuming normals),

$$\begin{aligned} X_{11}, \ldots, X_{1n_1} &\text{ i.i.d. } \mathrm{N}(\mu_1, \sigma_1), \\ &\vdots \qquad\qquad \vdots \\ X_{I1}, \ldots, X_{In_I} &\text{ i.i.d. } \mathrm{N}(\mu_I, \sigma_I), \end{aligned}$$

but we make the additional assumption: $\sigma_1 = \ldots = \sigma_I$.

# Statistical Model

Model:

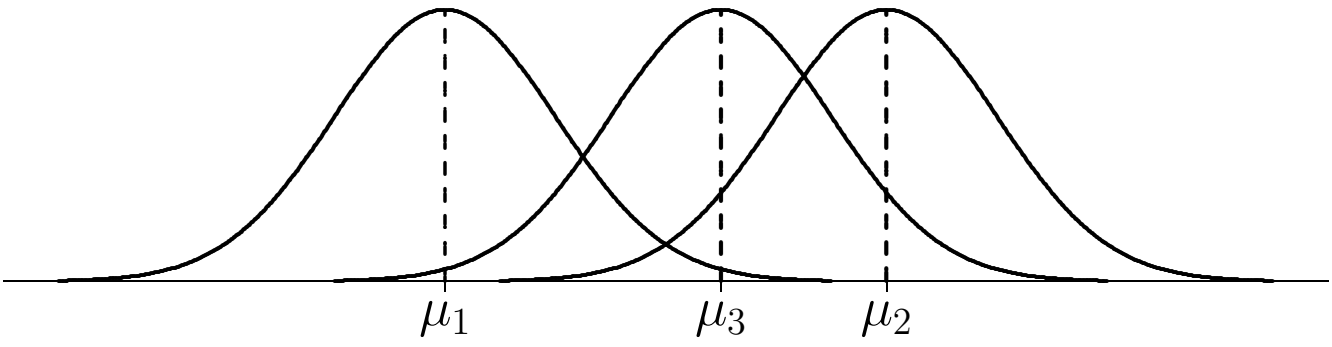
$$X_{ij} = \mu_i + \varepsilon_{ij}, \quad i = 1, \ldots, I; \; j = 1, \ldots, n_i,$$

where $\varepsilon_{ij}$'s are i.i.d. and $\sim \mathrm{N}(0, \sigma)$.

| $i$ | observations | | | |
|---|---|---|---|---|
| 1 | $X_{11} = \mu_1 + \varepsilon_{11}$ | $X_{12} = \mu_1 + \varepsilon_{12}$ | $\ldots$ | $X_{1n_1} = \mu_1 + \varepsilon_{1n_1}$ |
| 2 | $X_{21} = \mu_2 + \varepsilon_{21}$ | $X_{22} = \mu_2 + \varepsilon_{22}$ | $\ldots$ | $X_{2n_2} = \mu_2 + \varepsilon_{2n_2}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ldots$ | $\vdots$ |
| $I$ | $X_{I1} = \mu_I + \varepsilon_{I1}$ | $X_{I2} = \mu_I + \varepsilon_{I2}$ | $\ldots$ | $X_{In_I} = \mu_I + \varepsilon_{In_I}$ |

- parameters: $\mu_1, \ldots, \mu_I$ (group means) and $\sigma$ (common standard deviation of all $X$'s and $\varepsilon$'s),
- $\varepsilon_{ij} = X_{ij} - \mu_i$, i.e. the deviation of $X_{ij}$ from its mean $\Rightarrow$ $\varepsilon$'s interpreted as random errors / perturbations / noise,
- same model as on previous slide: $X_{ij} \sim \mathrm{N}(\mu_i, \sigma)$.

Normal distributions for 3 groups:



## Estimation

Rules and formulae for estimation of model parameters:

- group sample means as estimates for $\mu$'s:

$$\begin{aligned} \hat{\mu}_i &= \bar{X}_i \sim \mathrm{N}(\mu_i, \sigma/\sqrt{n_i}), \quad i = 1, \ldots, I, \\ \mathrm{SE}(\hat{\mu}_i) &= s_p/\sqrt{n_i}, \end{aligned}$$

- pooled sample variance as estimate for $\sigma^2$ (a weighted average of the group variances $s_i^2$):

$$\hat{\sigma}^2 = s_p^2 = \sum_i \frac{n_i - 1}{N - I} s_i^2 = \sum_{ij} \frac{(X_{ij} - \bar{X}_i)^2}{N - I} = \mathrm{SSE}/\mathrm{DFE},$$

$$\hat{\sigma} = s_p = \sqrt{s_p^2},$$

where

  - $\mathrm{SSE} = \sum_{ij}(X_{ij} - \bar{X}_i)^2$, the within-group sum of squares,
  - $\mathrm{DFE} = N - I = (n_1 - 1) + \ldots + (n_I - 1)$, the (within-group) degrees of freedom for $s_p^2$.

Confidence intervals for $\mu_i$ the "usual way", e.g.

$$(1-\alpha)\ \text{CI for } \mu_i: \ \hat{\mu}_i \pm t^* \,\mathrm{SE}(\hat{\mu}_i), \ t^* = t_{1-\alpha/2}(\mathrm{DFE}),$$

note: using the pooled standard deviation $s_p$ and its DF for the confidence intervals.

# Model Checking

Summary of model assumptions:

(1) all observations are independent,

(2) all observations are normally distributed,

(3) all observations have the same standard deviation (often called variance homogeneity or homoscedasticity),

(4) all observations within a group have the same mean.

Useful graphical displays:

- boxplots/stemplots/dotplots for all groups in same diagram (overview of data, assumptions (2) and (3)),
- normal probability plots for each of the groups (2).

Useful statistics:

- standard descriptive statistics for each group (3),
- normality tests for each of the groups (2).

Practical considerations:

- textbook (PSLS/IPS) guideline for when to "accept" assumption about equal standard deviations: ratio of largest to smallest standard deviation less than 2,
- tests of equal stand. deviations (in software): more sensitive than the 1-way ANOVA itself, but fine if non-significant,
- if all groups show the same non-normal pattern (e.g., skewness), transformation may be a solution,
- if higher group means are associated with higher standard deviation, transformation (log or $\sqrt{\cdot}$) may be a solution.

## Hypothesis and Test

Consider the overall group hypothesis: $\mathrm{H}_0$: $\mu_1 = \ldots = \mu_I$: (all groups equal, homogeneity between groups)

- alternative hypothesis $\mathrm{H}_a$: some $\mu$'s differ ("two-sided"),[4]
- test statistic calculated in several steps,
  - define group sum of squares: $\mathrm{SSG} = \sum_i n_i(\bar{X}_i - \bar{X})^2$ — a weighted sum of squared deviations between group means and the overall mean
  - define group degrees of freedom: $\mathrm{DFG} = I - 1$,
  - introduce group mean square (ratio between sum of squares and DF): $\mathrm{MSG} = \mathrm{SSG}/\mathrm{DFG}$,
  - finally, test statistic is: $F = \mathrm{MSG}/s_p^2$,
- some "motivations":
  - $F$ compares *variation between groups* with *variation within groups*,
  - nominator and denominator have similar forms: $F = \mathrm{MSG}/\mathrm{MSE}$, (because $s_p^2 = \mathrm{SSE}/\mathrm{DFE} = \mathrm{MSE}$),
- under $\mathrm{H}_0$: $F$-statistic $\sim F(\mathrm{DFG}, \mathrm{DFE})$, and large values are critical for $\mathrm{H}_0$ (one-sided test),
- $P$-value calculated as: $P = \mathrm{P}(F(\mathrm{DFG}, \mathrm{DFE}) > F_{\mathrm{obs}})$.

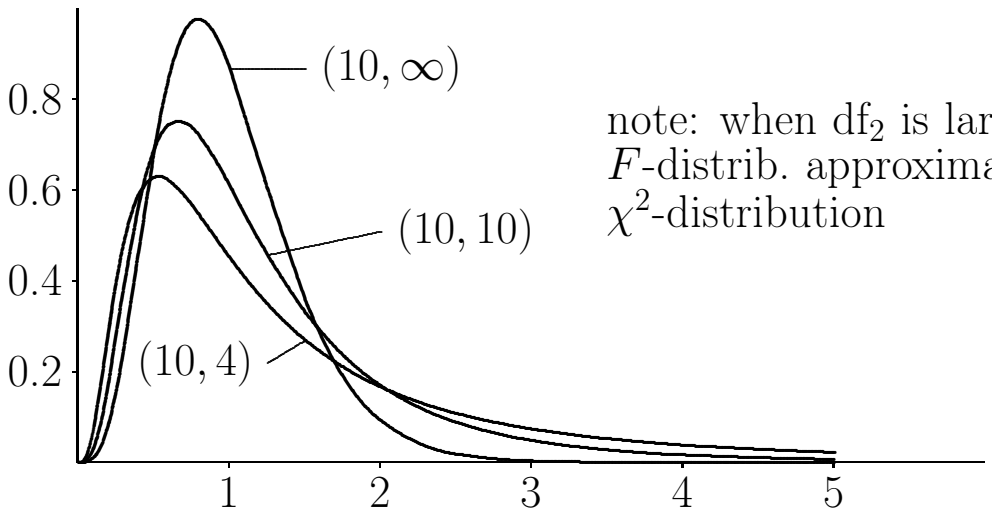
[4] It is a common misunderstanding that $\mathrm{H}_a$ implies all the $\mu$'s to be different, but the opposite of $\mathrm{H}_0$ is just that the means are not all the same.

# $F$-DISTRIBUTION

Another distribution — to be used for tests in normal models (that are more complex than one or two samples):

- $F$-distributions have two parameters, and we write $F(\mathrm{df}_1, \mathrm{df}_2)$ to indicate them:
  - $\mathrm{df}_1, \mathrm{df}_2$ are numbers in $\{1, 2, 3, \ldots\}$
  - called *numerator* ($\mathrm{df}_1$) and *denominator* ($\mathrm{df}_2$) "degrees of freedom", because $F$ variables are usually ratios,
  - given from the data, and not to be estimated,
  - the order of $\mathrm{df}_1$ and $\mathrm{df}_2$ is important, because $F(\mathrm{df}_1, \mathrm{df}_2) \neq F(\mathrm{df}_2, \mathrm{df}_1)$,
- distributions on $(0, \infty)$ — only positive values,
- right skewed; decreasing mean and standard deviation with increasing $\mathrm{df}_2$; percentiles in Table F/E of PSLS/IPS.

Some $F$ distributions:




note: when $\mathrm{df}_2$ is large ($\infty$), the $F$-distrib. approximates a scaled $\chi^2$-distribution

# ANOVA Table

Analysis of variance table
= convenient layout for summarizing the analysis:

- idea: split variation in the data into parts:

  total variation = variation between groups ("Groups")
  + variation within groups ("Error"),

- collect quantities related to each source of variation on separate line,
- provides good overview and eases computations,
- generalizes to models with more variables than one.

General ANOVA table:

| Source | Degrees of freedom | Sum of squares | Mean square | $F$ | $P$ |
|---|---|---|---|---|---|
| Groups | DFG $= I-1$ | SSG $= \sum_i n_i(\bar{X}_i - \bar{X})^2$ | MSG $=$ SSG/DFG | MSG/MSE | $\mathrm{P}(F \geq F_{\mathrm{obs}})$ |
| Error | DFE $= N-I$ | SSE $= \sum_{ij}(X_{ij} - \bar{X}_i)^2$ | MSE $=$ SSE/DFE | $F \sim F(\mathrm{DFG},\mathrm{DFE})$ | |
| Total | DFT $= N-1$ | SST $= \sum_{ij}(X_{ij} - \bar{X})^2$ | (MST $=$ SST/DFT) | | |

Notes:

- MST often omitted from table ($\neq$ MSG + MSE),
- always remember: $\hat{\sigma} = s_p = \sqrt{\mathrm{MSE}}$.

# Exercises 12.1, 12.9, 12.25

Exercise 12.1: (response, populations, $I$, $n_i$'s and $N$)

(a) response = tomato yield, $I = 4$ varieties, all $n_i = 12$, and $N = 48$,

(b) response = rate of attractiveness, $I = 5$ types of packaging, all $n_i = 40$, and $N = 200$,

(c) response = weight loss, $I = 3$ programs, all $n_i = 20$, and $N = 60$.

Exercise 12.9: (degrees of freedom and hypotheses) For all settings, the hypotheses are:

$$\mathrm{H}_0: \ \mu_1 = \mu_2 = \mu_3 = \mu_4, \ \ \mathrm{H}_a: \ \text{some } \mu\text{'s different},$$

where $I = 4, 5$ and 3, respectively.

(a) varieties (DF = 3), error (DF = 44), total (DF = 47), $F(3, 44)$,

(b) packagings (DF = 4), error (DF = 195), total (DF = 199), $F(4, 195)$,

(c) programs (DF = 2), error (DF = 57), total (DF = 59), $F(2, 57)$.

Exercise 12.25:

(a)

| Source | Degrees of freedom | Sum of squares | Mean square | $F$ |
|---|---|---|---|---|
| Groups | 3 | 104,855.87 | | |
| Error | 32 | 70,500.59 | | |
| Total | | | | |

(b) $\mathrm{H}_0: \ \mu_1 = \mu_2 = \mu_3 = \mu_4, \ \ \mathrm{H}_a: \ \text{some } \mu\text{'s different},$

(c) $F \sim F(3, 32)$ under $\mathrm{H}_0$, $\mathrm{P} = \mathrm{P}(F(3, 32) > 15.9) \ll 0.001 \Rightarrow$ reject $\mathrm{H}_0$; conclusion: there are some differences between groups,

(d) $s_p^2 = \mathrm{MSE} = 2203$, $s_p = \sqrt{2203} = 46.9$.

# Contrasts

Another type of null hypotheses:

- more specific than overall homogeneity of groups,
  (1) involves two particular groups, e.g. $H_0$: $\mu_D = \mu_S$, or
  (2) involves a linear combination of several group means, for example (Reading data): $\mu_B = (\mu_D + \mu_S)/2$,
- often (not necessarily) considered *after* test of overall $H_0$,
- decided prior to data collection and analysis!

Definition:
A *contrast* is a linear combination of group mean parameters of the form (in PSLS notation),

$$L = \sum_i c_i \, \mu_i,$$

where the $c_i$'s are known constants with sum 0 ($\sum_i c_i = 0$).

Examples from above: (1) $c_D = 1$, $c_S = -1$, $c_B = 0$, and (2) $c_B = 1$, $c_D = -0.5$ and $c_S = -0.5$.

Statistical inference about contrasts:

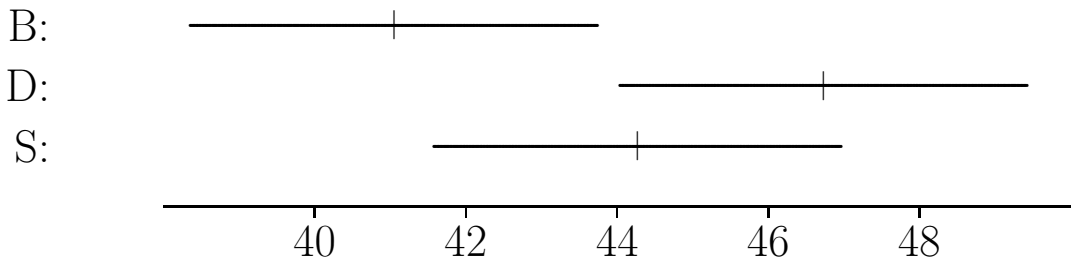
- estimate: $\hat{L} = \sum_i c_i \bar{X}_i$ (sample contrast),
- standard error: $\mathrm{SE}_{\hat{L}} = s_p \sqrt{\sum_i c_i^2 / n_i}$,
- $(1-\alpha)$ CI for $L$: $\hat{L} \pm t^* \, \mathrm{SE}_{\hat{L}}$, $t^* = t_{1-\alpha/2}(\mathrm{DFE})$,
- test of $H_0$: $L = 0$: $t = \hat{L}/\mathrm{SE}_{\hat{L}} \sim t(\mathrm{DFE})$ under $H_0$.

# Tips for Comparing and Presenting Groups

Group comparisons based on confidence intervals:

- based on test/CI for difference between parameters (next slide),
- but conclusions available from group CIs in 2/3 cases (see figure):

  Reading data example: 95% CIs for `post3`:




  * B vs. D: disjoint (non-overlapping) CIs $\Rightarrow$ signif. ($P < 0.05$),
  * D vs. S: estimate in another CI $\Rightarrow$ no signif. ($P > 0.05$),
  * B vs. S: need CI for difference ($\mu_B - \mu_S$) to assess signif.

  assumes independent estimates, unadjusted for multiple testing.

Significance letter coding[5]: from software or constructed manually,

- order group means from lowest to highest,
- designate letter $a$ to highest group + all groups not significantly different from it,
- designate letter $b$ to next group in the same way (but drop if same pattern as for $a$),
- continue through all groups,
- Reading data: (uncorrected 5% error) coding: $\text{B}^b$ $\text{S}^{ab}$ $\text{D}^a$.

[5] Meaning of letter codes: groups with same letter are *not* significantly different.

# Pairwise Comparisons

How to proceed after ANOVA and preplanned contrasts?

- if the overall $H_0$ is non-significant:
  no further analysis needed (relevant), report $\hat{\mu}$ and $P$,
- if the overall $H_0$ is significant:
  - for illustration: plot of $\hat{\mu}_i$'s with error bars,
  - for informal comparison of group levels — LSD:
    - **l**east **s**ignificant **d**ifference(s),
    - margin of error of CI's for pairwise differences $\mu_i - \mu_j$ based on $\bar{X}_i - \bar{X}_j$:

      $$\text{LSD}_{1-\alpha} = t^* \, s_p \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}, \quad t^* = t_{1-\alpha/2}(\text{DFE}),$$

    - most useful for *balanced* data (all $n_i$'s equal), because one LSD-value applies to all comparisons,
    - example (reading data, `post3`):

      $$\text{LSD}_{0.95} = t_{0.975}(63) \times 6.314 \times \sqrt{2/22} = 3.81,$$

    - interpretation: smallest distance between $\bar{X}_i$ and $\bar{X}_j$ so that CI for $\mu_i - \mu_j$ does not contain 0 ($\Rightarrow H_0 : \mu_i = \mu_j$ signif. at level $\alpha$, if preplanned),
    - same as pairwise 2-sample $t$-tests based on $s_p$ (Fisher),
  - for formal comparison of group levels we must take into account an increase in overall (simultaneous) error level for multiple unplanned[6] comparisons.

[6] To compare most extreme groups (means) is *not* a valid preplanned comparison!

## Bonferroni Method for Multiple Tests

Idea: If $A$ and $B$ are events, it always holds that

$$\mathrm{P}(A \text{ or } B) \leq \mathrm{P}(A) + \mathrm{P}(B).$$

In particular, in the context of performing several tests,

$$\mathrm{P}(\text{error in one or more tests}) \leq \text{ sum of error prob.}$$

Therefore, if we make $k$ tests/comparisons, we can achieve the *simultaneous* error probability *for all tests* to be $\leq \alpha$, by taking the error probability for each test equal to $\alpha/k$.

Adjustment of LSD method for $k$ preplanned tests:

$$\mathrm{LSD}_{1-\alpha/k} = t_{1-\alpha/(2k)}(\mathrm{DFE})\, s_p \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}.$$

Adjustment of LSD method for unplanned comparisons:
(suggested by the data)

- take $k =$ total no. of comparisons $= \binom{I}{2} = I(I-1)/2$,
- use above LSD-formula with that value of $k$.

Notes for Bonferroni method:

- is *conservative* (larger CI's and higher $P$-values),
- is flexible and applicable to many situations (not only 1-way ANOVA).

Tukey method, described in Chapter 26 of PSLS:

- acceptable method, less conservative than Bonferroni,
- method difficult to explain, and less flexible than Bonf.

## Summary of 1-Way ANOVA for Reading Scores

Statistical models (for `pre1` and `post3` variables):

$$X_{ij} = \mu_i + \varepsilon_{ij},$$

where $i = 1, 2, 3$ (Basal, DRTA, Strat), $j = 1, \ldots, 22$ (students), and $\varepsilon_{ij}$'s i.i.d. and $\sim \mathrm{N}(0, \sigma)$.

ANOVA tables:

| | | `pre1` | | | | `post3` | | | |
|---|---|---|---|---|---|---|---|---|---|
| Source | DF | SS | MS | $F$ | $P$ | SS | MS | $F$ | $P$ |
| Groups | 2 | 20.58 | 10.29 | 1.13 | 0.33 | 357.3 | 178.7 | 4.48 | 0.015 |
| Error | 63 | 572.45 | 9.09 | | | 2511.7 | 39.9 | | |
| Total | 65 | 593.03 | | | | 2869.0 | | | |

Hypothesis $\mathrm{H}_0$: $\mu_1 = \mu_2 = \mu_3$ (no differences), $\mathrm{H}_a$: not $\mathrm{H}_0$,
Test of $\mathrm{H}_0$ : $\rightarrow$ $F$-tests in table:
not significant for `pre1` but significant for `pre3`,

Estimation/Presentation (using $t^\star = t_{0.975}(63) = 2.00$):

| | `pre1` | `post3` | | |
|---|---|---|---|---|
| statistic | overall | Basal | DRTA | Strat |
| mean | 9.79 | 41.05 | 46.73 | 44.27 |
| std.dev. $s_p$ | $\sqrt{9.09} = 3.01$ | $\sqrt{39.9} = 6.31$ | | |
| SE(mean) | $s_p/\sqrt{66} = 0.37$ | $s_p/\sqrt{22} = 1.35$ | | |
| 95% CI | $\pm 0.74$ | $\pm 2.69$ | | |
| $\mathrm{LSD}_{0.95}$ | — | 3.81 | | |

Conclusions: (`pre1`): no differences before teaching,
(`post3`): no differences D/S or B/S; difference D/B.

## Sample Size for 1-way ANOVA

Based on desired precision (e.g., size of CI):

- CI could be for group mean, a difference between group means or a more general contrast,
- requires known (guessed) $\sigma$ and desired margin of error ($m$),
- general approach based on approximate 95% CI for a mean (or contrast) parameter $\mu$, based on its estimate $\hat{\mu}$:

$$\hat{\mu} \pm 2\,\mathrm{SE}(\hat{\mu})$$

— determine $\mathrm{SE}(\hat{\mu})$ as a function of $n$, and solve with respect to $n$ the equation: $m \geq 2\,\mathrm{SE}(\hat{\mu})$.

Based on power for $F$-test:

- requires known (guessed) $\sigma$ and group means $\mu_i$, plus significance level,
- calculation available in Minitab/Stata/R, and on websites.[7]

[7] The recommended website for sample size calculations is `http://homepage.stat.uiowa.edu/~rlenth/Power/`.

# Kruskal-Wallis Test

- Model: $I$ independent samples from different distributions:

$$\begin{array}{lll} X_{11}, \ldots, X_{1n_1} & \text{i.i.d.} & \text{with distribution Dist}_1, \\ \vdots & & \vdots \\ X_{I1}, \ldots, X_{In_I} & \text{i.i.d.} & \text{with distribution Dist}_I, \end{array}$$

- Hypotheses — two possibilities:
  - $\mathrm{H}_0$: $\text{Dist}_1 = \ldots = \text{Dist}_I$ (same distrib.), $\mathrm{H}_a$: not $\mathrm{H}_0$,[8]
  - assuming "$\text{Dist}_i = \text{Dist}_0 + \Delta_i$" (distributions all of the same shape, and differ only in positions $\Delta_i$), $\mathrm{H}_0$: $\Delta_i$'s=0 (corresponding to same medians) versus two-sided alternative $\mathrm{H}_a$,
- Test procedure:
  - rank all observations as if a single sample, and compute rank averages $\bar{R}_i$ for each group $i$,
  - test statistic: sum of squares for ranks,

$$H = const \times \textstyle\sum_i n_i(\bar{R}_i - \bar{R})^2, \quad \bar{R} = (N+1)/2,$$

    $\sim$ SSG in 1-way ANOVA for ranks,
  - under $\mathrm{H}_0$: distribution of $Y$ has no easy form, and software use different approximations for the $P$-value (based on the $\chi^2(I-1)$-distribution).

[8] More specific wording of $\mathrm{H}_a$: for some $(i, i')$, $P_i$ is systematically larger than $P_{i'}$; see Chapter 27 of PSLS.

## Summary Notes

Key words and concepts for 1-way ANOVA:

- comparison of multiple samples (SRS or i.i.d.), assumed normally distributed with separate means but same variance,
- estimation: sample means, pooled variance/standard deviation,
- model checking: normality in each sample, equal standard deviation rule,
- hypothesis and test: $F$-statistic and $F$-distribution, ANOVA table: sum of squares (SS), degrees of freedom (DF), mean square (MS),
- after ANOVA: significant test is followed by pairwise comparisons (or contrasts, not in course syllabus), Bonferroni adjustment for multiple comparisons,
- sample size calculations for 1-way ANOVA,
- nonparametric 1-way ANOVA: Kruskal-Wallis test.