Supplementary exercise 4.71 of IPS7e

A discrete probability distribution for the temperature (X) of the flame in a glass heating process. (As an aside, one might think that a continuous distribution would be more natural.) The probability function is given by the values in the table below. We note that the probability distribution is valid because the probabilities are ≥ 0 and sum to 1 across the possible outcomes.

Temperature x (°C)	540	545	550	555	560
Probability $p(x)$	0.10	0.25	0.30	0.25	0.10

(a) We calculate the mean temperature as (letting X denote a random variable from this distribution):

$$\mathrm{E}\,X = \sum_x x \cdot p(x) = 540 \cdot 0.1 + 545 \cdot 0.25 + 550 \cdot 0.3 + 555 \cdot 0.25 + 560 \cdot 0.1 = 550.$$

The distribution is (visibly) symmetrical around 550, so we could have guessed that the mean would equal 550.

We calculate the standard deviation for X in two steps: first the variance, and then the standard deviation:

$$\begin{aligned}
\mathrm{Var}\,X &= \sum_x (x - \mathrm{E}X)^2\, p(x) \\
&= (540-550)^2 \cdot 0.1 + (545-550)^2 \cdot 0.25 + (550-550)^2 \cdot 0.3 \\
&\quad + (555-550)^2 \cdot 0.25 + (560-550)^2 \cdot 0.1 \\
&= 10 + 6.25 + 0 + 6.25 + 10 = 32.5, \\
\mathrm{sd}\,X &= \sqrt{\mathrm{Var}X} = \sqrt{32.5} = 5.7.
\end{aligned}$$

(b) The new variable of interest is $T = X - 550$, i.e., a translation of the original variable X by the value (–550). We can either use the rules for a translation, or the general rules for linear transformation of a random variable.

$$\begin{aligned}
\mathrm{E}\,T &= \mathrm{E}X - 550 = 0 \quad \text{(i.e., we adjust the center by the translation)}, \\
\mathrm{sd}\,T &= \mathrm{sd}X = 5.7 \quad \text{(i.e., no impact on spread by a translation)}.
\end{aligned}$$

When using the formulae on slide 3L–16, you should set $b = 1$ and $a = (-550)$.

(c) Here the variable of interest is $Y = 1.8 \cdot X + 32$. We should use the mean and standard deviation rules for linear transformation of random variables (slide 3L–16), with $a = 32$ and $b = 1.8$.

$$\begin{aligned}
\mathrm{E}\,Y &= 32 + 1.8 \cdot \mathrm{E}X = 32 + 1.8 \cdot 550 = 1022, \\
\mathrm{sd}\,Y &= 1.8 \cdot \mathrm{sd}X = 1.8 \cdot 5.7 = 10.26 \approx 10.3.
\end{aligned}$$

We finally, on the next page, show Minitab commands to carry out these calculations after the distributions has been entered in suitable columns, labeled `x` and `p(x)`.

```
MTB > Name c1 "x"
MTB > Set 'x'
DATA>   1( 540 : 560 / 5 )1
DATA>   End.
MTB > name c2 "p(x)"
MTB > Set 'p(x)'
DATA>   1( 0.1 0.25 0.3 0.25 0.1 )1
DATA>   End.
MTB > Name C3 'meanx'
MTB > Let 'meanx' = 'x' * 'p(x)'
MTB > Sum 'meanx'.
MTB > Name C4 'varx'
MTB > Let 'varx' = ('x'-550)^2 * 'p(x)'
MTB > Sum 'varx'.
MTB > Name C5 'xF'
MTB > Let 'xF' = 1.8 * 'x' + 32
MTB > Name C6 'meanxF'
MTB > Let 'meanxF' = 'xF' * 'p(x)'
MTB > Sum 'meanxF'.
MTB > Name C7 'varxF'
MTB > Let 'varxF' = ('xF' - 1022)^2 * 'p(x)'
MTB > Sum 'varxF'.
MTB > Print 'x' 'p(x)' 'meanx' 'varx' 'xF' 'meanxF' 'varxF'.
```

Sum of meanx

Sum of meanx = 550

Sum of varx

Sum of varx = 32.5

Sum of meanxF

Sum of meanxF = 1022

Sum of varxF

Sum of varxF = 105.3

Data Display

Data

Row	x	p(x)	meanx	varx	xF	meanxF	varxF
1	540	0.10	54.00	10.00	1004	100.40	32.40
2	545	0.25	136.25	6.25	1013	253.25	20.25
3	550	0.30	165.00	0.00	1022	306.60	0.00
4	555	0.25	138.75	6.25	1031	257.75	20.25
5	560	0.10	56.00	10.00	1040	104.00	32.40

Note that the Minitab calculator cannot be used for a simple (scalar) calculation such as: $\sqrt{105.3} = 10.25$. You would either have to use the calculator built into Windows or another calculator.