

Supplementary exercise 6.95 of IPS7e

Measurements of radon concentrations by radon detectors placed in a test chamber. The true concentration is known to be 105 picocuries per liter, so the purpose of the experiment is to test the accuracy of the detectors.

Data: 12 readings, X_1, \dots, X_{12} , of home radon detectors when exposed to 105 pCi/l of radon.

Model: X_1, \dots, X_{12} are i.i.d. (a simple random sample) from $N(\mu, \sigma)$, a normal distribution with unknown mean μ and standard deviation σ . Generally speaking, we do not necessarily have to assume a normal distribution for the data, but we do this here to focus on the exercise questions. Also, a future exercise (7.64) will discuss the distribution of the data (and its compliance with a normal distribution). We will however, perhaps quite unrealistically, assume the standard deviation σ to be known: $\sigma = 9$. The average of the X 's, the sample mean, equals $\bar{X} = 104.133$.

- (a) The 95% confidence interval for μ , the mean reading in the chamber, is obtained by the formula,

$$\begin{aligned} 95\% \text{ CI for } \mu : \quad \bar{X} \pm 1.96 \cdot \sigma / \sqrt{n} &= 104.133 \pm 1.96 \cdot 9 / \sqrt{12} \\ &= 104.133 \pm 5.092 = (99.0, 109.2) \end{aligned}$$

- (b) We set up the hypotheses as

$$H_0 : \mu = 105 \quad \text{versus} \quad H_a : \mu \neq 105.$$

The z -statistic for H_0 against H_a is significant at the 5% level, if a 95% two-sided confidence interval for μ *does not* include the (target) value 105.

The interval from (a) however easily includes 105, so there is no significant evidence against the population mean being equal to 105. As shown by the Minitab output below, the P -value is 0.74. However, the focus of the exercise is on using the confidence interval only to reach the conclusion.

Note that the population studied consists of detectors of the given type placed in the same chamber (possibly extendable to another environment with the same radon concentration).

Finally, the Minitab command and output for both the confidence interval and the hypothesis test from the -1-Sample Z menu.

```
MTB > OneZ 'radon';
SUBC>   Sigma 9;
SUBC>   Test 105;
SUBC>   Confidence 95.0;
SUBC>   Alternative 0.
```

One-Sample Z: radon				
Descriptive Statistics				
N	Mean	StDev	SE Mean	95% CI for μ
12	104.13	9.40	2.60	(99.04, 109.23)
μ : mean of radon				
Known standard deviation = 9				
Test				
Null hypothesis		$H_0: \mu = 105$		
Alternative hypothesis		$H_1: \mu \neq 105$		
Z-Value	P-Value			
-0.33	0.739			