Index of 8-L

Practical Information

Lecture contents:

- 2-way (contingency) tables and chi-square tests[1], including Simpson's paradox and calculations for 2-way tables,[2]
- include in course: the distinction between the two models for 2-way tables,
- two additional topics:
  - multinomial distribution (in course curriculum),
  - Fisher's exact test (not in course curriculum),
- 2-sample proportions revisited (7L—6/7),
- guidelines on how to report statistics in papers/theses (detailed links and references at course webpage and Moodle site).

Schedule news:

- the quiz for Session 7 is on, until noon today Thursday,
- home assignment II to be returned on Monday, with the solution probably going on the website a bit earlier,
- home assignment III to be posted next week.

[1] PSLS 4e: Chapter 22; S: Section 11.2; IPS 7e: Sections 9.1-2.
[2] PSLS 4e: Chapter 5; IPS 7e: Section 2.5; not covered in S (note that S discusses Simpson's paradox for quantitative data only).

Data Example: Avadex for Mice

A clinical trial to assess a possible carcinogenic effect of Avadex (a fungicide).

- Data: control and treatment (Avadex in feed) groups of mice; number of mice with lung tumors recorded:

	Group		
Outcome	Avadex	control	Total
tumors	4	5	9
no tumors	12	74	86
Total	16	79	95

- Model: $X \sim \mathrm{B}(16, p_1)$ and $Y \sim \mathrm{B}(79, p_2)$, where X and Y are the counts of mice with tumors in the Avadex and control groups, respectively,
- Estimation:

$$\hat{p}_1 = 4/16 = 0.250, \quad \mathrm{SE}_{\hat{p}_1} = 0.108,$$
$$\hat{p}_2 = 5/79 = 0.063, \quad \mathrm{SE}_{\hat{p}_2} = 0.027,$$
$$\hat{p}_1 - \hat{p}_2 = 0.187, \quad \mathrm{SE}_{\hat{p}_1 - \hat{p}_2} = 0.112,$$

- Confidence intervals: (95%, plus four method)

$$p_1 : 0.30 \pm 0.20, \quad p_2 : 0.084 \pm 0.060, \quad p_1 - p_2 : 0.204 \pm 0.215.$$

- Hypotheses: H_0: $p_1 = p_2$ $(= p)$ vs. H_a: $p_1 \neq p_2$; estimate common p under H_0: $\hat{p} = 9/95 = 0.095$, and $\mathrm{SE}_{D_p} = \sqrt{\hat{p}(1-\hat{p})((1/16)+(1/79))} = 0.080$,
- Test: (classical, normal approximation) $z = (\hat{p}_1 - \hat{p}_2)/\mathrm{SE}_{D_p}$, $z_{\text{obs}} = 0.187/0.080 = 2.326$, which gives P-value $= 2 \cdot \mathrm{P}(Z > 2.326) = 0.020$.

Data Example: Health Habits of Students

A survey[3] obtained information on the levels of physical activity and consumptions of fruit — is there a link between these, or they are independent?

Data: responses obtained for 1184 college students:

	Physical activity			
Fruit consumption	low	moderate	vigorous	Total
low	69	206	294	569
medium	25	126	170	321
high	14	111	169	294
Total	108	443	633	1184

2 response variables, because none of the variables are fixed (known) in advance.

Descriptive statistics: (for response variables)

- Marginal distributions — looking at each variable separately: fruit consumption: 569/1184 = 48% low, 321/1184 = 27% medium, 294/1184 = 25% high; physical activity: 108/1184=9% low, 443/1184=37% moderate, 633/1184=53% vigorous,
- Conditional distributions — looking at one variable when the other is fixed: e.g. fruit consumption in low physical activity group: 69/108 = 64% low, 25/108 = 23% medium, 14/108=13% high.

[3] Data from Seo D-C et al. (2007), *Journal of Amererican College Health* 56, 187-197; also Example 9.8 of IPS 7e.

Data Example: Music and Wine Purchase

Experimental study on music's impact on wine purchase (number of bottles sold of categories French, Italian and other) in a supermarket[4] under different music conditions (none, French accordion music, Italian string music).

Data: 243 bottles sold categorized by wine type and music type:

	Music			
Wine	none	French	Italian	Total
French	30	39	30	99
Italian	11	1	19	31
other	43	35	35	113
Total	84	75	84	243

- 1 response variable — the type of wine purchased,
- 1 explanatory variable — the type of music played (controlled by the store) $\sim$ 3 separate time periods and therefore independent samples.

Descriptive statistics:

- Conditional distributions — proportions of wine sold for the three samples: e.g., Italian wine $\sim 19/84 = 23\%$ for Italian music, but only $1/75 = 1\%$ for French music,
- Marginal wine type distribution — pooled across music type: e.g., Italian wine $\sim 31/243 = 13\%$ of bottles sold.

[4] Study carried out in Northern Ireland in 1990s; Ryan et al. (1998), Proc. Nutrition Soc. 57, 169A; also Example 9.8 in IPS 6e.

Multinomial Distribution

Example: wine purchase when no music is played:

type of wine	French	Italian	Other	Total
count	30	11	43	84
symbol N_i	N_1	N_2	N_3	n
rel. frequency	0.357	0.131	0.519	1
symbol p_i	p_1	p_2	p_3	1

Multinomial Distribution:

$$(N_1, N_2, \ldots, N_q) \sim \text{multinomial}(n; p_1, p_2, \ldots, p_q)$$

where n is the total number of observations, q is the number of classes, and p_i is the (population) probability of each class (so that $p_1 + p_2 + \ldots + p_q = 1$), if

- mathematical definition:

$$\mathrm{P}(N_1 = n_1, \ldots, N_q = n_q) = \binom{n}{n_1 \ldots n_q} p_1^{n_1} \cdots p_q^{n_q}, \quad \text{where} \quad \binom{n}{n_1 \ldots n_q} = \frac{n!}{n_1! \cdots n_q!},$$

- conceptual definition ("multinomial setting"):
  - n trials; in each, one of q categories is observed,
  - independent trials, with same probabilities of the categories across all trials.

Note: As Multinomial$(n; p_1, p_2) = \mathrm{B}(n, p_1)$, the multinomial distribution extends the binomial to > 2 categories.

2-Way Tables: Notation

$I \times J$ table: n observations grouped (cross-classified) according to two criteria ($\sim$ categorical variables) A and B with I and J levels (categories), respectively:

Counts		$j \sim$ criterion B							
		1	2	$\dots$	j	$\dots$	$J-1$	J	sum
	1	N_{11}	N_{12}	$\dots$	N_{1j}	$\dots$	$N_{1,J-1}$	N_{1J}	$N_{1\cdot}$
	2	N_{21}	N_{22}	$\dots$	N_{2j}	$\dots$	$N_{2,J-1}$	N_{2J}	$N_{2\cdot}$
$i \sim$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
criterion	i	N_{i1}	N_{i2}	$\dots$	N_{ij}	$\dots$	$N_{i,J-1}$	N_{iJ}	$N_{i\cdot}$
A	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
	$I-1$	$N_{I-1,1}$	$N_{I-1,2}$	$\dots$	$N_{I-1,j}$	$\dots$	$N_{I-1,J-1}$	$N_{I-1,J}$	$N_{I-1,\cdot}$
	I	N_{I1}	N_{I2}	$\dots$	N_{Ij}	$\dots$	$N_{I,J-1}$	N_{IJ}	$N_{I\cdot}$
	sum	$N_{\cdot 1}$	$N_{\cdot 2}$	$\dots$	$N_{\cdot j}$	$\dots$	$N_{\cdot,J-1}$	$N_{\cdot J}$	n

Notes:

- always: $i \sim$ rows, and $I =$ number of rows,
  $j \sim$ columns, and $J =$ number of columns,
- each (i, j) combination corresponds to a table cell,
- the $N_{i\cdot}$'s are row totals, and the $N_{\cdot j}$'s are column totals,
- the textbook chapters are "notation-less", but double subscript notation will be used later on (for ANOVA).

2-Way Tables: Models and Estimation

Model I: **Independent multinomials**[5] over columns (or rows):

$$\begin{aligned}(N_{11}, \ldots, N_{I1}) &\sim \text{ multinomial } (N_{\cdot 1}; p_{11}, \ldots, p_{I1}),\\ &\vdots \\ (N_{1J}, \ldots, N_{IJ}) &\sim \text{ multinomial } (N_{\cdot J}; p_{1J}, \ldots, p_{IJ}),\end{aligned}$$

where p_{ij} = probability of group i in jth column, and all probability column sums equal 1,[6]

- Examples: Wine purchase, Avadex in mice,
- Assumptions: multinomial setting in each column, and independence between columns,
- Estimation: $\hat{p}_{ij} = N_{ij}/N_{\cdot j}$ — sample proportions within each column,
- Interpretation: one response variable (rows), one explanatory variable (columns).

Model II: a **Single multinomial**[7] on IJ classes:

$$(N_{11}, \ldots, N_{ij}, \ldots, N_{IJ}) \sim \text{ multinomial } (n; p_{11}, \ldots, p_{ij}, \ldots, p_{IJ}),$$

where p_{ij} = probability of group (cell) (i, j), and all probabilities sum to 1,[8]

- Example: Health habits,
- Assumptions: multinomial setting for table (IJ cells),
- Estimation: $\hat{p}_{ij} = N_{ij}/n$ — table sample proportions,
- Interpretation: 2 response variables (rows and columns).

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[5] IPS: model for comparing several populations or independent SRSs.
[6] In symbols, for every column j ($j = 1, \ldots, J$): $\sum_i p_{ij} = 1$, or written out: $p_{1j} + \ldots p_{Ij} = 1$.
[7] IPS: model for examining independence or for a single SRS.
[8] In symbols, $\sum_{ij} p_{ij} = 1$, or written out: $p_{11} + p_{12} + \ldots + p_{1J} + p_{21} + \ldots + p_{2J} + \ldots + p_{IJ} = 1$.

2-Way Tables: Hypotheses

Model I: Independent multinomials over columns:

- Hypothesis H_0: homogeneity among columns (same distribution in all columns):

$$H_0 : p_{ij} = p_{i\cdot} \text{ for all } j, \quad \text{and } H_a : \text{ not } H_0,$$

  and H_0 corresponds to using the marginal distribution across columns (row totals),
- Estimation under H_0: $\hat{p}_{i\cdot} = N_{i\cdot}/n$,
- Expected value of cell (i, j) under H_0: e_{ij} = row total $\times$ column total / overall total.

Model II: a Single multinomial on IJ classes:

- Hypothesis H_0: independence between row and column classification:

$$H_0 : p_{ij} = p_{i\cdot}\, p_{\cdot j} \text{ for all } i \text{ and } j, \quad \text{and } H_a : \text{ not } H_0,$$

  and H_0 corresponds to using the marginal distribution across both rows and columns,
- Interpretation:

$$\begin{aligned} p_{ij} &= \text{P}(\text{row} = i \text{ and column} = j) \\ &= (\text{independence}) \ \text{P}(\text{row} = i) \times \text{P}(\text{column} = j) = p_{i\cdot}\, p_{\cdot j}, \end{aligned}$$

- Estimation under H_0: $\hat{p}_{i\cdot} = N_{i\cdot}/n$, and $\hat{p}_{\cdot j} = N_{\cdot j}/n$,
- Expected value of cell (i, j) under H_0: e_{ij} = row total $\times$ column total / overall total.

2-Way Tables: Test

Result: In both of the models I and II, we test H_0 (homogeneity or independence) by the (Pearson chi-square) statistic,

$$X^2 = \sum_{i,j} \frac{(N_{ij} - e_{ij})^2}{e_{ij}} = \sum_{i,j} \frac{(\text{observed}_{ij} - \text{expected}_{ij})^2}{\text{expected}_{ij}}$$

$$\sim \chi^2 \text{ distribution(df) under } H_0,$$

where $\text{df} = (I-1) \cdot (J-1)$.[9]

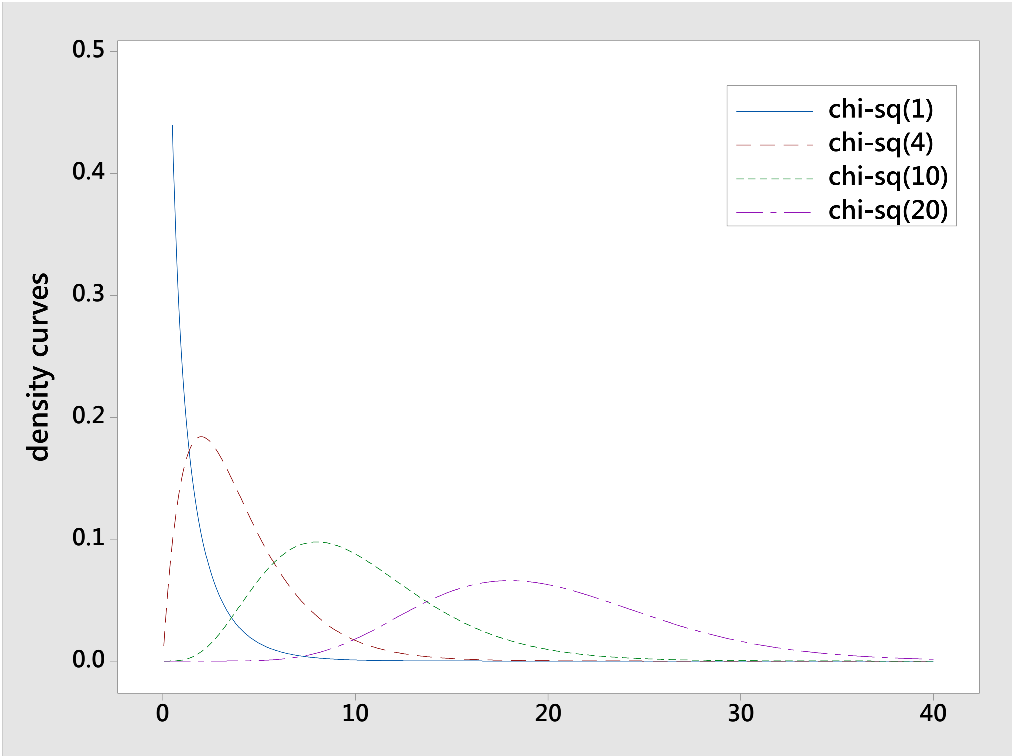
Notes:

- same statistic, but different models and hypotheses $\Rightarrow$ different interpretations!
- one-sided test: only large values are critical, and $P = \mathrm{P}(\chi^2(\text{df}) > X^2_{\text{obs}})$,
- distribution of X^2 under H_0 is approximate and best for large n; guidelines for use:
  - $e_{ij} > 1$ in all cells (i, j), and
  - $e_{ij} > 5$ in at least 80% of cells (i, j),
- other test for H_0 exist and may be better...

[9] Technical note: the degrees of freedom can in both models be calculated as number of free parameters in model minus number of free parameters under H_0.

χ^2 Distributions

- "chi-square" distributions,
- a new distribution — to be used for test statistics in tables of categorical data (not for modelling), as the reference distributions for X^2 (or χ^2) statistics,
- a single parameter df (df $= 1, 2, 3, \ldots$), the degrees of freedom (determined by the statistical design),
- denoted $\chi^2(\text{df})$ to indicate the degrees of freedom,
- distribution on $(0, \infty)$ — only positive values,
- mean $=$ df, standard deviation $= \sqrt{2\text{df}}$,
- tail probabilities: software or tables.[10]



[10] PSLS Table D; S Table 5; IPS Table F.

Music and Wine Purchase — Analysis

- Model: three independent multinomial distributions on 3 classes,
- Hypothesis H_0: same distribution of wine purchases for all types of music,
- Test: see table below for calculation of X^2:
  - $\mathrm{df} = (3-1)\cdot(3-1) = 4$,
  - $X^2_{\mathrm{obs}} = 18.28$, $P = \mathrm{P}(\chi^2(4) > 18.28) = 0.0011$,
- Conclusion: very clear evidence against H_0 $\Rightarrow$ conclude that wine purchase depends on the music played,
- Presentation and estimation: show the observed proportions separately for the three samples (because of the rejection of H_0).

Calculation of X^2:

count (e_{ij})	Music			
Wine	none	French	Italian	Total
French	30 (34.2)	39 (30.6)	30 (34.2)	99
Italian	11 (10.7)	1 (9.6)	19 (10.7)	31
other	43 (39.1)	35 (34.9)	35 (39.1)	113
Total	84	75	84	243

$$X^2 = \frac{(30-34.2)^2}{34.2} + \frac{(39-30.6)^2}{30.6} + \ldots + \frac{(35-39.1)^2}{39.1} = 18.28.$$

Health Habits — Analysis

- Model: a single multinomial distribution on 9 classes,
- Hypothesis H_0: independence between levels of fruit consumption and exercise,
- Test: see table below for calculation of X^2:
  * $\text{df} = (3-1) \cdot (3-1) = 4$,
  * $X^2_{\text{obs}} = 14.15$, $P = \text{P}(\chi^2(4) > 14.15) = 0.007$,
- Conclusion: very clear evidence against $H_0 \Rightarrow$ conclude that dependence exists between fruit consumption and exercise levels,
- Presentation and estimation: conditional distributions for fruit consumption given physical activity, or conversely; cells of major interest: (low, low) and (high, low).

Calculation of X^2:

count (e_{ij})	Physical activity			
Fruit	low	moderate	vigorous	Total
low	69 (51.9)	206 (212.9)	294 (304.2)	569
medium	25 (29.3)	126 (120.1)	170 (171.6)	321
high	14 (26.8)	111 (110.0)	169 (157.2)	294
Total	108	443	633	1184

$$X^2 = \frac{(69-51.9)^2}{51.9} + \frac{(206-212.9)^2}{212.9} + \ldots + \frac{(169-157.2)^2}{157.2} = 14.15.$$

2×2 Tables

2×2 tables = special case of 2-way tables:

- simplest case, and very common in practice,
- some special relations for the chi-square statistic:
  - $X^2 = z^2$, where z is the statistic for comparing two binomial distributions,
    $\Rightarrow$ methods equivalent (same P-value), and guideline for use of X^2 also applies to z!
  - easier computational formula for X^2:
    $$X^2 = \frac{(N_{11}N_{22} - N_{12}N_{21})^2}{N_{1\cdot}N_{2\cdot}N_{\cdot 1}N_{\cdot 2}} \times n$$
- an almost endless selection of methods / procedures:
  - alternative tests for same hypothesis:
    - Fisher's exact test (next slide),
    - continuity-correction for the X^2 statistic,[11]
  - other measures for comparison of probabilities than differences: relative risk and odds-ratio (epi course),
  - tests for other hypotheses (e.g. McNemar's test, 7L–10) ...,
  - other statistics (e.g. kappa values) ...,
- simple advice: use your common sense and use only procedures that you understand (the rationale and assumptions behind).

[11] In order to get better approximations by χ^2-distributions; recommended to use Fisher's test instead if this a concern.

Fisher's Exact Test

Fisher's exact test for 2 independent binomial distributions:

- ∘ test of null hypothesis H_0: $p_1 = p_2$ against one- or two-sided alternative H_a,
- ∘ test "statistic" = observed table (one cell of table),
- ∘ idea: compare observed table with other tables that have the same margins (row and column sums),[12]
- ∘ P-value for one-sided H_a = sum of table probabilities for tables more indicative for H_a than for H_0,
- ∘ P-value for two-sided H_a = twice the smallest one-sided P, or the sum of table probabilities less than for observed table (most commonly used: Minitab/Stata/R).

Why / When use Fisher's exact test?

+ works also if χ^2-approximation not good ⇒ recommended if X^2-guidelines violated,
+ allows one-sided alternative hypothesis (just as z-tests),

− requires software, and for large samples computing time may be very long,

∗ applies also to single multinomial model and its test of independence,

∗ a version of the test exists also for larger tables than 2×2.

[12] (technical) Under H_0, tables with the same margins ∼ hypergeometric distribution, see example on next page.

Avadex Example: Fisher's Exact Test

Data table
(with expected values)
and general notation:

Outcome	Avadex	control	Total		Avadex	control	Total
tumors	4 (1.5*)	5 (7.5)	9		a	b	K
no tumors	12 (14.5)	74 (74.5)	86		c	d	$N-K$
Total	16	79	95		n	$N-n$	N

* expected value $< 5 \Rightarrow$ conditions for X^2-test violated

Idea: under H_0, we consider all table margins as fixed and focus only on the distribution of the $K = 9$ tumor cases among the two samples,

- similar to sampling of n elements from a finite population of size N of which K are "cases" $\rightarrow$ a hypergeometric distribution (N, K, n, a) for the number of cases a in the sample.

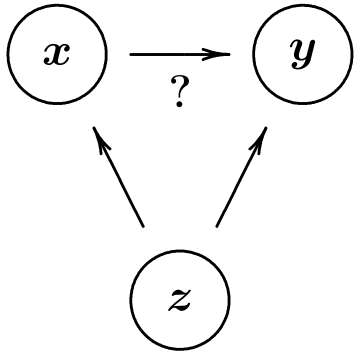
Scenario	Cell counts a	b	c	d	Prob. under H_0	One/two-sided tail prob. one- ($>$)	one- ($<$)	two- ($\neq$)
1	0	9	16	70	.175		.175	
2	1	8	15	71	.335		.335	
3	2	7	14	72	.296		.296	
4	3	6	13	73	.132		.132	
5	4	5	12	74	.035	.035	.035	.035
6	5	4	11	75	.006	.006		.006
7	6	3	10	76	.001	.001		.001
8	7	2	9	77	.000	.000		.000
9	8	1	8	78	.000	.000		.000
10	9	0	7	79	.000	.000		.000
P-value = sum of tail probabilities						.041	.994	.041

- note: the two-sided P-value is computed by adding up probabilities for tables with probability $\leq$ observed table; here, it equals one of the one-sided P-values.

Simpson's Paradox

= an extreme affect of ignoring a lurking variable,

– at closer look, not really a paradox at all.



Example: Punctuality of airlines
(AA=Alaska Airlines, AW=America West;
IPS7e Supplementary Exercise 9.19):

Summary 2-way tables:

	All airports		Los Angeles		Phoenix	
On time/airline	AA	AW	AA	AW	AA	AW
on time	718	5534	497	694	221	4840
delayed	74	532	62	117	12	415
sum	792	6066	559	811	233	5255
prop. delayed	0.093	0.088	0.111	0.144	0.052	0.079

x = airline (AA/AW)
y = on time (yes/no)
z = airport (Los Angeles/Phoenix)

Comments and conclusions:

- Simpson's paradox:
  - ∗ overall, airline is AW better (has less delays) than AA,
  - ∗ in both airports, airline AA is better than AW,
- explanation: airport Los Angeles has more flights delayed, and AA has more flights at this airport,
- conclusion: "paradox" may happen whenever both effects $z \to x$ and $z \to y$ are strong.

How to Report Statistics in Scientific Papers?

The last two decades has seen much stronger focus on appropriate conduct and reporting of statistical analysis in the published (peer-reviewed) literature, due to

- greater awareness of problems/issues, in particular in (human) health sciences,
- much larger variety of statistical methods becoming accessible through (variety of) statistical software,
- development of systematic review and meta-analysis (where studies reported inappropriately cannot be included),
- debate on philosophical issues around statistical analysis, in particular statistical testing (to be discussed in Session 12 of the course),
- general interest in establishing more strict guidelines for scientific research.

Many guidelines for specific study types exist, covering planning, execution, analysis and interpretation, in particular through the EQUATOR (Enhancing the QUAlity and Transparency Of health Research) network. Scientific journals increasingly require compliance with relevant guideline(s).

General statistical reporting guidelines:

- "original" (old, still useful): Bailar & Mosteller, 1988 (I),
- more recent: Lang & Altman, 2013 (II).

Statistical Reporting Guidelines I

Developed by the "International Committee of Medical Journal Editors"[13]:

1. Describe statistical methods with enough detail to enable a knowledgeable reader with access to the original data to verify the reported results.
2. When possible, quantify findings and present them with appropriate indicators of measurement error or uncertainty (such as confidence intervals).
3. Avoid sole reliance on statistical hypothesis testing, such as the use of P values, which fails to convey important quantitative information.
4. Discuss eligibility of experimental subjects.
5. Give details about randomization.
6. Describe the methods for, and success of, any blinding of observations.
7. Report treatment complications.
8. Give numbers of observations (and give the experimental unit).
9. Report losses to observation (such as dropouts from a clinical trial).
10. References for study design and statistical methods should be to standard works (with pages stated) when possible rather than to papers where designs or methods were originally reported.
11. Specify any general-use computer programs used.
12. Put general descriptions of statistical methods in the Methods section. When data are summarized in the Results section, specify the statistical methods used to analyze them.
13. Restrict tables and figures to those needed to explain the argument of the paper and to assess its support. Use graphs as an alternative to tables with many entries; do not duplicate data in graphs and tables.
14. Avoid non-technical uses of technical terms in statistics, such as "random" (which implies a randomizing device), "normal", "significant", "correlation", and "sample".
15. Define statistical terms, abbreviations, and most symbols.

[13] Bailar, J.C. & Mosteller, F. (1988), *Annals of Internal Medicine* 108, 266−73.

Statistical Reporting Guidelines II

Developed by two prominent statistics authors, but without formal consultation process (as for other guidelines)[14], and aims to cover "basic" statistical analyses and methods.

- First guiding principle: statements $(1, 2, 3, 10, 11, 15)$ from Bailar & Mosteller.
- Second guiding principle: provide enough detail that the results can be incorporated into other analyses.
- Reporting statistical methods — split into: preliminary, primary and supplementary analyses ($\Rightarrow$ researchers need to categorize their analyses and in particular identify their primary analyses, for which (among other things) a purpose must be described).
- Reporting of statistical results — split into different types of outcomes and analyses, e.g. for hypothesis testing (selected topics/items):
  - state the hypothesis being tested; report whether the test was one- or two-tailed (justify the use of one-tailed tests),
  - identify the variables in the analysis and the name of the test used,
  - if possible, identify the minimum difference considered to be clinically important; for equivalence and non-inferiority studies, report the largest difference between groups that will still be accepted as indicating biological equivalence,
  - confirm that the assumptions of the test were met by the data,
  - report the alpha level that defines statistical significance,
  - at least for primary outcomes, report a measure of precision, such as the 95% confidence interval (do NOT use the standard error to indicate the precision of an estimate),
  - although not preferred to confidence intervals, if desired, P values should be reported as equalities when possible and to one or two decimal places; the smallest P value that need be reported is $P < 0.001$, save in studies of genetic associations,
  - report whether and how many adjustments were made for multiple statistical comparisons.

[14] Lang T, Altman D (2013). Statistical Analyses and Methods in the Published Literature: the SAMPL guidelines, http://www.equator-network.org/reporting-guidelines/sampl/.

Summary Notes

Key words and concepts:

- two-way table (of counts), 2×2-table (and larger),
- marginal and conditional distributions in two-way tables, Simpsons paradox,
- multinomial distribution for counts in > 2 categories,
- 2 models/hypotheses for two-way tables of counts:
  - independent multinomial distributions (for comparing several populations/samples), with hypothesis of homogeneity,
  - single multinomial distribution for entire table (for single population/sample), with hypothesis of independence,
- X^2-test statistic:
  - computation from observed and expected counts,
  - reference χ^2-distribution, and its degrees of freedom,
  - guideline for use of X^2-test (and its χ^2 reference distribution),
  - relationship with 2-sample z-test (for 2×2-table),
- Fisher's exact test for sparse tables (not in course curriculum).
- statistical reporting is under increasing scientific scrutiny, and it is recommended to rely on reporting guidelines for any publications.