Supplementary exercise 5.7 of IPS7e

The number of accidents (X) has mean $\mu = 2.2$ and standard deviation $\sigma = 1.4$. It has a discrete distribution, which is therefore definitely not normal (a Poisson distribution would be one possibility). We consider the average number of accidents during 52 weeks, denoted as $\bar{X}$.

(a) The central limit theorem (CLT) says that the average ($\bar{X}$) is approximately normally distributed with mean $E\bar{X} = \mu = 2.2$ and standard deviation $\text{sd}\bar{X} = \sigma/\sqrt{52} = 1.4/7.21 = 0.194$.

(b) Using the approximate normal distribution for $\bar{X}$, we get

$$P(\bar{X} < 2) = P\left(\frac{\bar{X} - 2.2}{0.194} < \frac{2 - 2.2}{0.194}\right) = P(Z < -1.03) = 0.15,$$

using statistical table or software.

(c) Fewer than 100 accidents per year corresponds to less than $100/52 = 1.923$ accidents per week. We compute in the same way as above: $P(\bar{X} < 1.923) = P(Z < -1.43) = 0.077$. The difference to the probability in (b) is substantial although the difference between 2 and 1.923 accidents per week seems small; relative to the standard deviation for $\bar{X}$ it is however quite considerable.

Finally some illustrations for direct calculation in Minitab.