Supplementary exercise 5.54 of IPS7e

Rules for approximations of distributions.

(a) We use the rule from slide 4L−10. The situation is a simple random sample of size 25 from a finite population of size 75. The sampling is without replacement, so it is not a binomial setting. Furthermore, the sampled fraction of the population is 25/75=0.3333, so the guideline for the approximation: that the sampled fraction must be no larger than 0.05, or that the population must be at least 20 times as large as the sample, is clearly violated.

(b) We use the rule from slide 5L−22. The situation is a simple random sample of size 500 from a huge population, so that the binomial distribution is a good approximation. Therefore, our model is a binomial $\mathrm{B}(500, p)$. With $p = 0.002$, the guideline for using the normal approximation for calculations in the $\mathrm{B}(n, p)$: that $n\,p\,(1–p) > 10$, is clearly violated because $500 \cdot 0.002 \cdot 0.998 = 0.998$. It may be helpful to display this binomial distribution graphically to see why it cannot be approximated well by a normal distribution; the Minitab command below does this (from the Probability Distribution Plot menu, View Single, and after choosing the binomial distribution and inserting the parameters).

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DPlot;
 Distribution;
  Binomial 500 .002.
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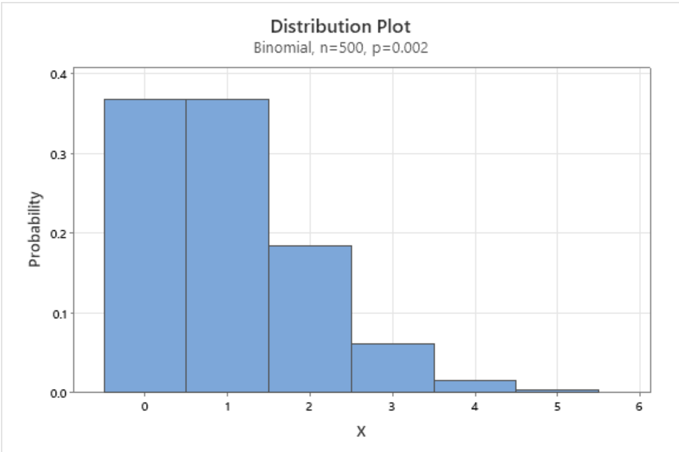

The distribution is clearly right-skewed, due to it being squeezed up against its lower bound at 0. Generally speaking, binomial distibutions tend to become very skewed when the probability parameter p is extreme in the interval (0,1), i.e. very close to 0 or 1. If p was actually equal to 0 or 1, there would not be a real distribution to consider (because only one outcome would be possible).