Supplementary exercise 8.103 of IPS7e

Sample size calculation to obtain a margin of error of 0.10 in a 95% confidence interval for the difference between two proportions, based on two independent binomial samples.

Denote the counts of the two samples by X and Y, so that $X \sim B(n, p_1)$ and $Y \sim B(n, p_2)$. The classical, approximate 95% CI for the difference $p_1 - p_2$ is (from slide 7L−15):

$$D \pm z^* \cdot \mathrm{SE}(D) = \hat{p}_1 - \hat{p}_2 \pm 1.96 \cdot \mathrm{SE}(D),$$

where $\hat{p}_i$ is the sample proportion in sample i, $i = 1, 2$, and $\mathrm{SE}(D) = \sqrt{\hat{p}_1(1-\hat{p}_1)/n + \hat{p}_2(1-\hat{p}_2)/n}$. The $\mathrm{SE}(D)$ (and therefore also the margin of error) is largest if both $\hat{p}_1$ and $\hat{p}_2$ are equal to 0.5. We will therefore get a conservative estimate of the required sample size if we work with both $\hat{p} = 0.5$.

To get a margin of error of at most 0.10, we need to solve

$$\begin{aligned} 0.1 &\geq 1.96 \cdot \sqrt{0.5 \cdot 0.5/n + 0.5 \cdot 0.5/n} \approx 2/\sqrt{2n}, \\ \text{or} \quad 2n &\geq (2/0.1)^2 = 400, \\ \text{or} \quad n &\geq 200 \quad \text{(or 193, if working with 1.96 rather than the rounded-off value of 2).} \end{aligned}$$

We need at least 200 subjects in each group to get a margin of error less than 0.1. This estimate can be substantially too large if the proportions in the two populations are far from 0.5.

From this calculation, we deduce that the general formula for a conservative sample size determination, at a desired margin of error m and a critical value z^*, for comparison of two independent proportions, is

$$n \geq (z^*/m)^2/2.$$

This formula is not in the VHM 801 textbooks. The expression for n is seen to be twice of the expression for a single sample. One valid approach to this (conservative) sample size calculation is therefore to double the value for n from a one-sample calculation (which can be done in Minitab).

The formula/approach is conservative because in practice one would rarely expect both probabilities to be very close to 0.5. If we instead used guessed values (p_1 and p_2) for the two proportions, the formula would become

$$n \geq (z^*/m)^2 \cdot (p_1(1-p_1) + p_2(1-p_2)).$$

Regardless of which version of the formula we use, we will need to verify when n has been determined that the conditions for use of the classical confidence interval are met. This may in particular be a problem if at least one of the two proportions is close to 0 or 1. If the conditions are not met, the calculation of the margin of error can be redone with the chosen n and the "plus four" method, and if the resulting margin of error is $> m$, the sample size must be increased.