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## PRACTICAL INFORMATION

### Today's lecture:

- follow-up/review from Session 5 based on Exercise 6.95 and 5L–19,
- inference for continuous data, **without assumption of known  $\sigma$** :
  - \* a new distribution: the ***t*-distribution(s)**,<sup>1</sup>
- lots of examples (data sets!) and practice with statistical inference,
- **inference for one and two samples** (continuous data):<sup>2</sup>
  - \* a single sample (no assumption of known  $\sigma$ ),
  - \* two independent samples,
  - \* two dependent (paired) samples.

### Scheduling notes:

- home assignment is due anytime today (my mailbox/office, or submitted electronically on Moodle); next home assignment will be posted on October 12,
- lab session 6 tomorrow *and* on October 16 (duplicated),
- midterm and final exam dates set (listed at webpage).

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<sup>1</sup> PSLS 4e: Chapter 17; S: Section 7.4; IPS 7e: Section 7.1.

<sup>2</sup> PSLS 4e: Chapters 17-18; S: Chapters 8-9 (parts); IPS 7e: Sections 7.1-2.

## OUTLINE OF STATISTICAL ANALYSIS (REVISITED)

- Data description,
- Statistical model,
- Estimation of model's unknown parameter(s),
  - \* incl. confidence intervals and/or standard errors,<sup>3</sup>
- Model check:
  - \* comparison of the observed distribution and assumed theoretical distribution (using estimated parameters),
  - \* **methods**: graphical (plots) or numerical (tests),
  - \* if model is deemed unsatisfactory, **start over with new model**,
- Hypothesis testing:
  - \* set up **null hypothesis**  $H_0$  (model simplification) and **alternative hypothesis**  $H_a$ ,
  - \* **test statistic** and associated  **$P$ -value** summarize our confidence **against null hypothesis**, which we may **reject** (low  $P$ ) or **not reject** (high  $P$ ),
- Conclusion / Presentation:
  - \* summary of test results,
  - \* illustrations of the **implications** of the final model, e.g. prediction.

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<sup>3</sup> Recall, that the standard error (SE) is the standard deviation in the distribution of the estimate, and thus an indication of the estimate's precision.

## 1-SAMPLE ESTIMATION

**Data:** sample  $X_1, \dots, X_n$  of size  $n$  from some distribution with **unknown mean**  $\mu$  and **unknown standard deviation**  $\sigma$  (and variance  $\sigma^2$ ). More specifically, we assume

- the  $X$ 's are i.i.d. (independent, identically distributed),
- $EX_i = \mu$  and  $\text{sd}X_i = \sigma$  for all  $X$ 's.

For **estimation of  $\sigma$**  we use the sample standard deviation:

$$\hat{\sigma} = s \quad (= \sqrt{s^2}) \quad \text{and} \quad \hat{\sigma}^2 = s^2 = \sum_{i=1}^n (X_i - \bar{X})^2 / (n-1),$$

and  $s^2$  is an **unbiased** estimate of  $\sigma^2$ , explaining why we use  $(n-1)$  for  $s^2$ .<sup>4</sup>

**Summary** of terminology and estimates for a single sample:

Name	Estimate	Parameter	Properties
<b>sample mean</b>	$\bar{X}$	$\mu$	unbiased
sample variance	$s^2$	$\sigma^2$	unbiased
<b>sample standard deviation</b>	$s$	$\sigma$	biased, natural
(sample variance of mean)	$s^2/n$	$\sigma_{\bar{X}}^2 = \sigma^2/n$	unbiased
<b>standard error of mean</b>	$s/\sqrt{n}$	$\sigma_{\bar{X}} = \sigma/\sqrt{n}$	biased, natural

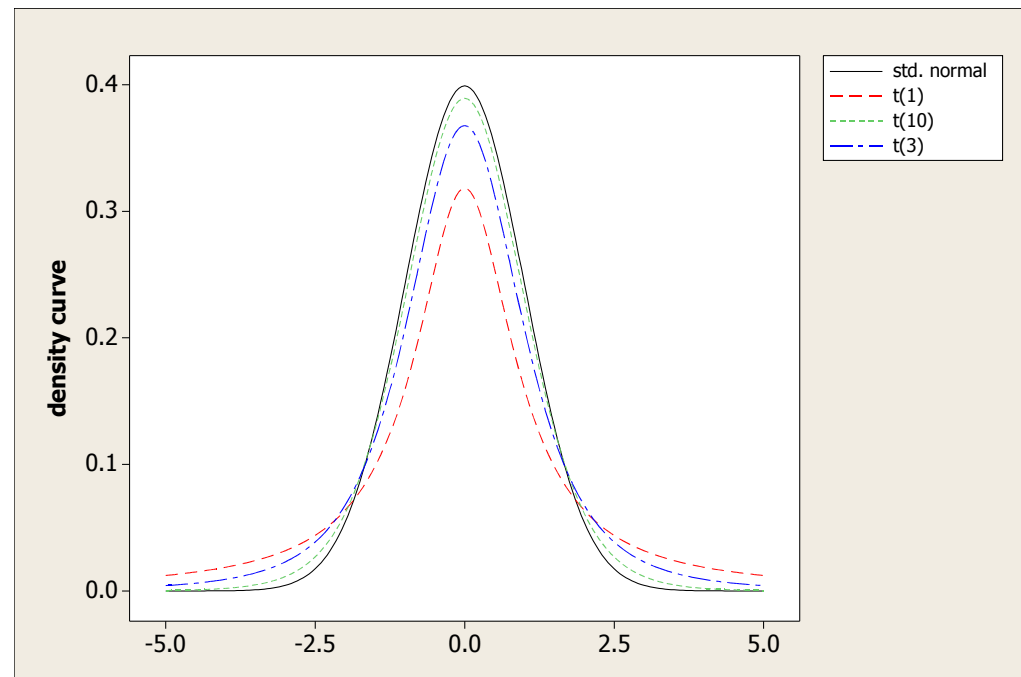
In addition,  $\bar{X} \sim N(\mu, \sigma_{\bar{X}})$

- exactly — if the  $X$ 's are normally distributed,
- approximately (when  $n$  is “large”) — always! (by the CLT)

<sup>4</sup> We skip over the (small) mathematical calculation showing that  $s^2$  is unbiased.

## (“STUDENT’S”) $t$ DISTRIBUTION

- new distribution(s) — to be used **not for modelling** but for inference in a normal distribution model **when  $\sigma$  is estimated from the data**, as the reference distribution for  $t$  test statistics (next slide),
- has a **single parameter  $r$**  (or “df”):
  - \*  $r = 1, 2, 3, \dots$
  - \* called “**degrees of freedom**” (explanation to follow),
  - \* given from the data, and not to be estimated.
- denoted  $t(r)$  to indicate degrees of freedom,
- distribution on  $(-\infty, \infty) \Rightarrow$  positive and negative values,
- symmetric around zero, almost “bell-shaped” but with heavier tails than  $N(0,1)$  ( $\Rightarrow$  positive kurtosis),
- when  $r$  is large:  
 $t(r) \approx N(0, 1)$ , see graph.



## 1-SAMPLE NORMAL DISTRIBUTION INFERENCE

- **Data:**  $X_1, \dots, X_n$  ( $n$  = number of observations).
- **Model:** observations are a sample (i.i.d.) from  $N(\mu, \sigma)$ , where  $\mu$  and  $\sigma$  are unknown parameters.
- **Estimation:**  $\hat{\mu} = \bar{X}$  and  $\hat{\sigma} = s$ .
- **Distribution of estimates:**

$$\hat{\mu} = \bar{X} \sim N(\mu, \sigma/\sqrt{n}), \quad s_{\bar{X}} = s/\sqrt{n},$$

$$(\bar{X} - \mu)/s_{\bar{X}} \sim t(n-1),$$

note that **degrees of freedom (df)** =  $n-1$ ,
- **Confidence interval** with confidence level  $1-\alpha$ :
$$\mu : \bar{X} \pm t^* s_{\bar{X}} = \bar{X} \pm t^* s/\sqrt{n},$$

where  $t^*$  is  $(1-\frac{\alpha}{2})$ -percentile of a  $t(n-1)$  distribution,<sup>5</sup>
- **Test** of  $H_0: \mu = \mu_0$  against alternative  $H_a$ :
  - \* **test statistic:**  $t = (\bar{X} - \mu_0)/s_{\bar{X}} = (\bar{X} - \mu_0)/(s/\sqrt{n})$ ,
  - \* **P-value** from  $t$  distribution with  $df = n-1$ :
    - $H_a: \mu \neq \mu_0$ :  $P = 2 \times P(t(df) \geq |t_{\text{obs}}|)$ ,
    - $H_a: \mu > \mu_0$ :  $P = P(t(df) \geq t_{\text{obs}})$ , and  $H_a: \mu < \mu_0$ :  $P = P(t(df) \leq t_{\text{obs}})$ ,
- note strong **similarities** with  $z$ -based procedures.

<sup>5</sup> In notation,  $t^* = t_{1-\alpha/2}(df)$  in the  $t(df)$  distribution; see Table C of PSLS, Table 3 of S, Table D of IPS.

## EXAMPLE: HUMAN BODY TEMPERATURE

Example 14.9 of PSLS 4e, Example p. 139 of S:

○ **Data:** 130 measurements<sup>6</sup> of body temperature in °F of healthy adults:  $X_1, \dots, X_{130}$  ( $n = 130$ ).

○ **Model:** a sample (i.i.d.) from  $N(\mu, \sigma)$ .

○ **Estimation:**  $\hat{\mu} = \bar{X} = 98.25$  and  $\hat{\sigma} = s = 0.733$ .

○ **Confidence interval** with confidence level 95% ( $\alpha = 0.05$ ):

$$\mu : \bar{X} \pm t^* s_{\bar{X}} = 98.25 \pm 1.98 \times 0.733 / \sqrt{130} = 98.25 \pm 0.13 = (98.12, 98.38),$$

using  $t^* = t_{.975}(129) = 1.9785$  from Minitab,

○ **Test** of  $H_0: \mu = 98.6$  against alternative  $H_a: \mu \neq 98.6$  (“classical” body temp.):

\* **test statistic:**  $t = \frac{\bar{X} - 98.6}{s_{\bar{X}}} = \frac{98.25 - 98.6}{0.733 / \sqrt{130}} = -5.45,$

\* **P-value** from  $t$  distribution with  $df = n - 1 = 129$ :

$$P = 2 \times P(t(129) \geq 5.45) < 0.000001 \text{ (Minitab)}$$

\* **Conclusion:** strong evidence to say that average body temperature is different, actually lower, than the “classical” reference value.

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<sup>6</sup> Constructed data for pedagogical purposes (Shoemaker (1996), *Journal of Statistics Education* 4) based on a real study from 1992 whose purpose it was to evaluate the well-established average body temperature of 37.0°C or 98.6°F, Mackowiak et al. (1992), *Journal of the American Medical Association* 268, 1578-1580.

## HOW TO FIND PERCENTILES AND *P*-VALUES

### Recall that

- the  $p\%$  percentile has  $p\%$  of the distribution below, and  $(100-p)\%$  above, where  $(100-p)\%$  is the tail probability,<sup>7</sup>
- *P*-values are typically determined from tail probabilities  $P(t \geq t_{\text{obs}})$  in standard distributions, e.g.  $N(0, 1)$  or  $t(\text{df})$ .

### Methods to determine percentiles or tail probabilities:

- **Minitab**: Probability Distribution Plot-View Probability menu with Shaded Area defined by Probability or X Value for percentiles and probabilities, respectively,<sup>8</sup>
- **Stata**: functions normal, invnormal, ttail, invttail; similar functions in **R**,<sup>9</sup>
- **statistical tables** with values for some confidence/error levels,
  - \* what to do, if **df is not in table**? — use largest value below df  
⇒ conservative analysis (larger CIs and *P*-values),
  - \* what to do, if  **$t_{\text{obs}}$ -value is not in table**? — find closest (“critical”) values in table, for example  $t_1 < t_{\text{obs}} < t_2$ , and use the relations

$$P(t \geq t_2) < P(t \geq t_{\text{obs}}) < P(t \geq t_1).$$

<sup>7</sup> In some statistical tables, the  $p\%$  percentile is the **critical value** for a one-tailed test with  $\alpha = (100-p)\%$ .

<sup>8</sup> Alternatively, the non-graphical Calc-Probability Distributions menu with Inverse Cumulative Probability or Cumulative Probability, respectively.

<sup>9</sup> **R** functions: pnorm, qnorm, pt, qt.

## EXERCISES 7.50 AND 7.52

### Exercise 7.50:

**Percentiles/critical values** for confidence intervals for population mean (with unknown population standard deviation):

- (a)  $n = 20$  and  $C = 95\%$ :  $\alpha = 0.05$  and  $t^* = t_{1-\alpha/2}(n-1) = t_{.975}(19) = 2.093$ .
- (b)  $n = 30$  and  $C = 90\%$ :  $\alpha = 0.10$  and  $t^* = t_{1-\alpha/2}(n-1) = t_{.95}(29) = 1.699$ .
- (c)  $n = 50$  and  $C = 80\%$ :  $\alpha = 0.20$  and  $t^* = t_{1-\alpha/2}(n-1) = t_{.90}(49) \approx t_{.90}(40) = 1.303$ ,  
— a conservative value (exact value (software): 1.299).

### Exercise 7.52:

Testing  $H_0: \mu = 0$  against  $H_a: \mu > 0$  based on a sample of 15 observations. Observed  $t$ -value is  $t_{\text{obs}} = 2.15$ .

- (a) **degrees of freedom** =  $15 - 1 = 14$ .
- (b-d) **percentiles** from  $t(14)$ :

$$t_{0.975}(14) = 2.145 < 2.15 < 2.264 = t_{0.98}(14),$$

with **right-tail probabilities** of 0.025 and 0.02, respectively; therefore,  
for  $P = P(t(14) \geq t_{\text{obs}})$  we have:  **$0.02 < P < 0.025$** ,

- (f) the test is **significant** at the 5% level, but **not significant** at the 1% level.
- (g) **exact  $P$ -value** (software) is  $P = 0.02476$ .

## INFERENCE FOR NON-NORMAL DATA

If data show strong / moderate **deviations from normality**:

- **remove outliers** (if any), and see if it helps,
- try to **transform the data**, and see if situation is better for transformed data,
  - \* **many transformations exist**: log and square-root common for right-skewed data,
  - \* results at transformed scale should always be **backtransformed** to original scale:
    - backtransformed means  $\sim$  **medians** in original data,<sup>10</sup>
    - for CI's: backtransform both endpoints,
- **nonparametric** statistical methods with no distributional assumptions (next lecture),
- some procedures based on the normal distribution are **robust** (or resistant), that is, work reasonably well even if assumptions are (mildly) violated (saved by the normality of  $\bar{X}$  in the CLT!):
  - \* difficult to know exactly what is okay and when — some guidelines:<sup>11</sup>
    - $n < 15$ : only if data close to normal (okay!),
    - $15 \leq n < 40$ : ok unless strong skewness or outliers,
    - $40 \leq n$ : also ok for clearly skewed distributions, but beware of strong outliers.

<sup>10</sup> It is more difficult to get means and SEs on original scale, and potentially less meaningful.

<sup>11</sup> Guidelines for  $t$ -distribution procedures from PSLS/IPS texts; the discussion in S is less detailed/satisfactory: assume normality or  $n > 30$ , in my view a very debatable guideline!. If  $\sigma$  is known, the inference is even less affected by non-normality, because it is the procedures involving  $s^2$  that rely most strongly on the normality assumption.

## 2 PAIRED SAMPLES

Paired (matched, correlated) samples/observations:

- **Data:**  $(X_1, Y_1) \dots, (X_n, Y_n)$  independent observation pairs:
  - \* typical **examples of pairs:**
    - **same individual:** left–right, before–after,
    - **different individuals:** twins, related or similar individuals,
  - \* in **experimental design terminology:**
    - pairs  $\sim$  blocks (of size 2),
    - observations within pairs  $\sim$  different treatments,
  - \* **purpose of pairs:** reduce variability and impact of other (lurking) factors,
- **Model and Analysis:**
  - \* usually work with **differences:**  $D_i = Y_i - X_i$ , (ratios  $Y_i/X_i$  or other functions also possible),
  - \* assume  $D_1, \dots, D_n$  sample from distribution  $(\mu_D, \sigma_D)$ , where
$$\mu_D = ED_i = EY_i - EX_i,$$
  - \* **hypothesis  $H_0$ :**  $\mu_D = 0 \sim$  no difference between (means of)  $X$ s and  $Y$ s,
  - \* **all methods for single sample inference apply!**

## 2 PAIRED SAMPLES: VISUAL RECEPTIVE FIELD

- **Data:** Neural activity (# spikes/sec) for a monkey's neuron in 9 recordings of both Response (R) and Spontaneous activity (SA), (PSLS Example 27.6)

Recording ( $i$ )	SA ( $X_i$ )	R ( $Y_i$ )	Difference ( $D_i$ )
1	2.5	16.7	14.2
2	7.5	20.0	12.5
...	...	...	...
9	17.5	10.0	-7.5

- **Model:** one sample (i.i.d.) of differences  $D_1, \dots, D_9$  assumed to follow  $N(\mu_D, \sigma_D)$ , where  $\mu_D = \mu_Y - \mu_X$  is the parameter of principal interest,
- **Estimation:**  $\hat{\mu}_D = \bar{D} = 16.87$ ,  $\hat{\sigma}_D = s_D = 16.40$ ,
- **95% Confidence interval** for  $\mu_D$ :

$$\bar{D} \pm t^* s_D / \sqrt{n} = 16.87 \pm 2.306 \cdot 16.40 / \sqrt{9} = 16.87 \pm 12.61,$$

- **Test** of  $H_0: \mu_D = 0$  ( $\sim \mu_Y = \mu_X$ ) against alternative  $H_a: \mu_D > 0$  ( $\sim \mu_Y > \mu_X$ ):

- \* **test statistic:**  $t = \frac{\bar{D} - 0}{s_D / \sqrt{n}} = \frac{16.87}{16.40 / \sqrt{9}} = 3.09$ ,

- \* **P-value** from  $t$  distribution with  $df=8$ :  $P = P(t(8) > 3.09) = 0.007$ ,<sup>12</sup>

- \* **conclusion:** clearly significant difference between neural activity at SA and R, and higher R activity.

<sup>12</sup> The  $t(8)$ -distribution table has  $t_{.99}(8) = 2.896$  and  $t_{.995}(8) = 3.355$ , from which we would get:  $0.005 < P < 0.01$ .

## 2 INDEPENDENT SAMPLES – INTRODUCTION

- **Data:**  
 $X_1, \dots, X_{n_1} \sim$  first sample, of size  $n_1$ ,  
 $Y_1, \dots, Y_{n_2} \sim$  second sample, of size  $n_2$ .
- **Model:** all observations independent, and the  $X$ 's and  $Y$ 's are samples from separate distributions,
- typical **example:** treatment and control groups, e.g. study on parasite burdens in Lithuanian calves,
- how to **distinguish from paired samples?**
  - \* not necessarily the same number of observations (i.e., maybe  $n_1 \neq n_2$ ),
  - \* no relation between  $X_1$  and  $Y_1$ ,  $X_2$  and  $Y_2$ , etc.
  - \* the  $X$ 's are interchangeable (“replications”), and the same for the  $Y$ 's.

**Overview of inference for mean difference  $\mu_1 - \mu_2$ ,** based on normal distributions:

- **assumptions:** normal distributions  $N(\mu_1, \sigma_1)$  and  $N(\mu_2, \sigma_2)$  for the two samples, with all parameters unknown,
- **slightly different procedures** depending on whether
  - (1)  $\sigma_1 \neq \sigma_2$  (general situation<sup>13</sup>).
  - (2)  $\sigma_1 = \sigma_2$  (simplest; often unrealistic assumption).<sup>14</sup>

<sup>13</sup> Without a specific assumption about the  $\sigma$ 's: we could have  $\sigma_1 = \sigma_2$  or  $\sigma_1 \neq \sigma_2$ .

<sup>14</sup> Not part of VHM 801 syllabus; PLS and S texts avoid this (“pooled variance”) method.

### EXERCISE 7.40

Identify **statistical design** as either (1) single sample, (2) matched pairs (paired sample) or (3) two independent samples:

- (a) **two independent samples**, because different groups of children and only one score from each child; the before versus after element is not part of the data collection,
- (b) **two paired samples**, because two scores are collected from each child, in random order; nor is the before versus after element part of the data collection here,
- (c) **one sample**, because only one sample (of 20 measurements) is taken,
- (d) **two independent samples**, because there is no connection between the measurements taken with the new and old method.

**What about** a slight variation of the design where 10 samples are taken from the specimen and each is analyzed with both the new and old method?

That would be **two paired samples**.

## 2 INDEPENDENT SAMPLES – EQUAL VARIANCES

- **Models:** 1<sup>st</sup> sample:  $N(\mu_1, \sigma_1)$ , 2<sup>nd</sup> sample:  $N(\mu_2, \sigma_2)$ ,
- **assume**  $\sigma_1 = \sigma_2 = \sigma$ , based on judgement<sup>15</sup> or test<sup>16</sup>,
- **Estimation of means:**  $\hat{\mu}_1 = \bar{X} \sim N(\mu_1, \sigma/\sqrt{n_1})$ ,  $\hat{\mu}_2 = \bar{Y} \sim N(\mu_2, \sigma/\sqrt{n_2})$
- **estimation of  $\sigma$**   
 (from  $s_1$  and  $s_2$  in  $X$ - and  $Y$ -samples):
 
$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \quad \text{and} \quad \hat{\sigma} = s = \sqrt{s^2},$$
 — “pooled”  $s^2$ : a weighted average of  $s_1^2$  and  $s_2^2$ ,
- **standard error** of mean difference:  $s_{\bar{X}-\bar{Y}} = s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ ,
- **degrees of freedom:**  $df = (n_1 - 1) + (n_2 - 1) = n_1 + n_2 - 2$ ,
- **Confidence interval** of level  $(1 - \alpha)$  for  $\mu_1 - \mu_2$ :
 
$$\mu_1 - \mu_2 : \bar{X} - \bar{Y} \pm t^* s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \quad t^* = t_{1-\alpha/2}(df),$$
- **Test of  $H_0: \mu_1 = \mu_2$**  against altern.  $H_a$ , using **test statistic:**  $t = (\bar{X} - \bar{Y}) / \left( s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$ ,
  - \*  $P$ -value from  $t$  distribution, e.g. computed as:  
 $H_a: \mu_1 \neq \mu_2: P = 2 \times P(t(df) > |t_{\text{obs}}|)$ , or  $H_a: \mu_1 > \mu_2: P = P(t(df) > t_{\text{obs}})$ ,
- note **similarities** with 1-sample procedures.

<sup>15</sup> PSLS/IPS guideline for assuming equal standard deviations:  $s_{\max}/s_{\min} \leq 2$ .

<sup>16</sup> Variance tests (especially Bartlett’s test) are overly sensitive to non-normality.

## 2 INDEPENDENT SAMPLES – GENERAL

Similar procedure – changes in  $s_{\bar{X}-\bar{Y}}$  and df:

- no assumption of  $\sigma_1 = \sigma_2$ :  $\Rightarrow$  more general procedure (also for when  $\sigma_1 \approx \sigma_2$ ),
- Estimation of means and standard deviations: separately for each sample:  $\bar{X}, s_1, \bar{Y}, s_2$ ,
- standard error of mean difference:  $s_{\bar{X}-\bar{Y}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ ,
- degrees of freedom – two approaches:
  - \* conservative (too low/“safe”) df:  $\min(n_1 - 1, n_2 - 1)$ ,
  - \* approximate with “terrible” formulas<sup>17</sup>, but the approximations are generally considered good,
- Confidence interval of level  $(1 - \alpha)$  for  $\mu_1 - \mu_2$ :

$$\mu_1 - \mu_2 : \bar{X} - \bar{Y} \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \quad t^* = t_{1-\alpha/2}(\text{df})$$

- Test of  $H_0: \mu_1 = \mu_2$  against altern.  $H_a$ , using test statistic:  $t = (\bar{X} - \bar{Y}) / \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ ,
  - \*  $P$ -values from  $t$  distribution in the same way, e.g.
    - $H_a: \mu_1 \neq \mu_2$ :  $P = 2 \times P(t(\text{df}) > |t_{\text{obs}}|)$ ,
    - $H_a: \mu_1 > \mu_2$ :  $P = P(t(\text{df}) > t_{\text{obs}})$ .

<sup>17</sup> Minitab uses Satterthwaite method, Stata/R use Welch method; both ok to use.

## 2 INDEPENDENT SAMPLES: PARASITE DATA

- **Data:**  $n_1 = 10$  and  $n_2 = 9$  parasite counts of calves on infected ( $X$ ) and safe ( $Y$ ) pasture,
- **Model:** 2 independent samples (i.i.d.) from  $N(\mu_1, \sigma_1)$  and  $N(\mu_2, \sigma_2)$ , respectively,
- **Estimation:**  $\hat{\mu}_1 = \bar{X} = 51.2$ ,  $\hat{\sigma}_1 = s_1 = 24.0$ , and  $\hat{\mu}_2 = \bar{Y} = 23.8$ ,  $\hat{\sigma}_2 = s_2 = 17.6$ ,
- some difference in estimated standard deviations, so we are **not** going to assume that  $\sigma_1 = \sigma_2$ ,
- **Confidence interval** with confidence level 95%:

$$\begin{aligned}\mu_1 - \mu_2 &: \bar{X} - \bar{Y} \pm t^* \sqrt{s_1^2/n_1 + s_2^2/n_2}, \\ &= 27.4 \pm 2.12 \sqrt{24.0^2/10 + 17.6^2/9} = 27.4 \pm 20.3,\end{aligned}$$

where  $t^* = t_{.975}(16) = 2.12$  (df computed by software),

- **Test** of  $H_0: \mu_1 = \mu_2$  against alternative  $H_a: \mu_1 \neq \mu_2$ :

$$* \text{ test statistic: } t = \frac{\bar{X} - \bar{Y}}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = 2.86,$$

- \* approximate  $P$ -value from  $t$  distribution with  $df = 16$ :

$$P = 2 \times P(t(df) > 2.86) = 0.011.$$

- \* **conclusion:** significant difference between parasite burdens on infected and safe pastures; that is, parasite levels are lower on safe pasture.

## SUMMARY NOTES

### Key words and concepts:

- statistical inference for **1 sample** (quantitative outcome) on a normal distribution with **unknown parameters** (mean and standard deviation):
  - \* sample standard deviation ( $s$ ) as estimate of population standard deviation ( $\sigma$ ), standard error (for sample mean),
  - \*  $t$ -distribution, degrees of freedom,
  - \* formulae for  $t$ -based confidence interval and  $t$ -test,
- finding/approximating  $P$ -values and critical values ( $t^*$ ),
- **designs** involving 1 and 2 samples (any distribution):
  - \* 1-sample, 2 independent samples, 2 paired (dependent, correlated) samples,
  - \* 2 paired samples  $\rightarrow$  1-sample for differences,
- choice of method/assumption for **2 independent normal** distribution samples:
  - \* **equal variances assumed**: pooled variance estimate, ratio of standard deviations  $\geq 2$  rule,
  - \* **no variance assumption**: df determined by software.