

Supplementary exercises 6.11 and 6.12 of IPS7e

Exercise 6.11

Weights (in *kg*) of male runners. Let X_1, \dots, X_{24} denote the 24 weights, i.e. $n = 24$. We assume that the observations are i.i.d. (independent and identically distributed) with mean μ and standard deviation σ ; as usual, they are the population parameters corresponding to the population these runners could be representative for. We also assume (quite unrealistically) that σ is known to be $\sigma = 4.5$ *kg*. The average of the X 's is: $\bar{X} = 61.79$ *kg*.

- (a) The standard error for \bar{X} , which we could also refer to as the standard deviation in the distribution of \bar{X} (i.e., $\sigma_{\bar{X}}$), is computed as:

$$SE = \sigma_{\bar{X}} = \sigma / \sqrt{24} = 4.5 / \sqrt{24} = 0.919.$$

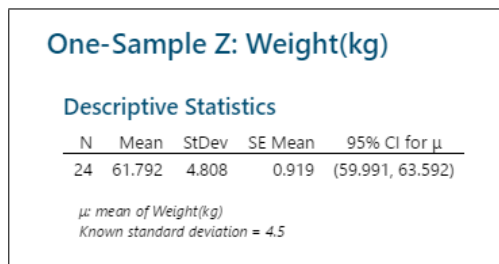
- (b) For a 95% confidence level, we use $z^* = 1.96$:

$$95\% \text{ CI: } \bar{X} \pm z^* \cdot SE = 61.79 \pm 1.96 \cdot 0.919 = 61.79 \pm 1.80 = (59.99, 63.59).$$

The confidence interval is entirely, and quite substantially, below 65 *kg*, so we can be pretty confident that the average weight in the population is below 65 *kg*. One could compute confidence intervals with higher confidence levels to see how strong that confidence is, but 95% confidence may be sufficient in itself.

Minitab command and output for this calculation (from the “Basic Statistics – 1-Sample Z” menu):

```
OneZ 'Weight(kg)';  
Sigma 4.5;  
Confidence 95.0;  
Alternative 0.
```



One-Sample Z: Weight(kg)				
Descriptive Statistics				
N	Mean	StDev	SE Mean	95% CI for μ
24	61.792	4.808	0.919	(59.991, 63.592)

μ : mean of Weight(kg)
Known standard deviation = 4.5

Exercise 6.12

We explore the impact of recording the weights instead in pounds (where each *kg* corresponds to 2.2 pounds). First, and as requested in the exercise, we use the formulae for linear transformation to $Y = a + bX$ (in this case, a scaling with $b = 2.2$ and $a = 0$) to convert the results from 6.11 to the new scale. Finally, we confirm our findings by scaling the individual observations and redoing the calculation, in Minitab.

- (a) By the formula for the mean after linear transformation,

$$EY = a + bEX = 2.2 \cdot EX = 2.2 \cdot 61.79 = 135.94.$$

- (b) The corresponding formula for the standard deviation is:

$$sdY = bsdX = 2.2 \cdot 4.5 = 9.9.$$

- (c) As the two distribution values in the CI formula were both multiplied by 2.2, the same happens to the bounds of the CI; alternatively, we use the same formula:

$$95\% \text{ CI: } \bar{Y} \pm z^* \cdot SE = 135.94 \pm 1.96 \cdot 9.9 / \sqrt{24} = 135.94 \pm 3.96 = (131.98, 139.90).$$

Minitab command and output for this calculation:

```
Name C3 'Weight(pounds)'  
Let 'Weight(pounds)' = 'Weight(kg)' * 2.2  
OneZ 'Weight(pounds)';  
Sigma 9.9;  
Confidence 95.0;  
Alternative 0.
```

One-Sample Z: Weight(pounds)

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
24	135.94	10.58	2.02	(131.98, 139.90)

μ : mean of Weight(pounds)
Known standard deviation = 9.9