Supplementary exercises 6.11 and 6.12 of IPS7e

Exercise 6.11
Weights (in kg) of male runners. Let $X_1, \dots, X_{24}$ denote the 24 weights, i.e. $n = 24$. We assume that the observations are i.i.d. (independent and identically distributed) with mean μ and standard deviation σ; as usual, they are the population parameters corresponding to the population these runners could be representative for. We also assume (quite unrealistically) that σ is known to be $\sigma = 4.5\ kg$. The average of the X's is: $\bar{X} = 61.79\ kg$.

(a) The standard error for $\bar{X}$, which we could also refer to as the standard deviation in the distribution of $\bar{X}$ (i.e., $\sigma_{\bar{X}}$), is computed as:

$$\mathrm{SE} = \sigma_{\bar{X}} = \sigma/\sqrt{24} = 4.5/\sqrt{24} = 0.919.$$

(b) For a 95% confidence level, we use $z^* = 1.96$:

$$95\%\ \mathrm{CI}: \ \bar{X} \pm z^* \cdot \mathrm{SE} = 61.79 \pm 1.96 \cdot 0.919 = 61.79 \pm 1.80 = (59.99, 63.59).$$

The confidence interval is entirely, and quite substantially, below 65 kg, so we can be pretty confident that the average weight in the population is below 65 kg. One could compute confidence intervals with higher confidence levels to see how strong that confidence is, but 95% confidence may be sufficient in itself.

Minitab command and output for this calculation (from the "Basic Statistics – 1-Sample Z" menu):

```
OneZ 'Weight(kg)';
 Sigma 4.5;
 Confidence 95.0;
 Alternative 0.
```

One-Sample Z: Weight(kg)

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
24	61.792	4.808	0.919	(59.991, 63.592)

μ: mean of Weight(kg)
Known standard deviation = 4.5

Exercise 6.12
We explore the impact of recording the weights instead in pounds (where each kg corresponds to 2.2 pounds). First, and as requested in the exercise, we use the formulae for linear transformation to $Y = a + bX$ (in this case, a scaling with $b = 2.2$ and $a = 0$) to convert the results from 6.11 to the new scale. Finally, we confirm our findings by scaling the individual observations and redoing the calculation, in Minitab.

(a) By the formula for the mean after linear transformation,

$$\mathrm{E}Y = a + b\,\mathrm{E}X = 2.2 \cdot \mathrm{E}X = 2.2 \cdot 61.79 = 135.94.$$

(b) The corresponding formula for the standard deviation is:

$$\mathrm{sd}Y = b\,\mathrm{sd}X = 2.2 \cdot 4.5 = 9.9.$$

(c) As the two distribution values in the CI formula were both multiplied by 2.2, the same happens to the bounds of the CI; alternatively, we use the same formula:

$$95\%\ \mathrm{CI}: \ \bar{Y} \pm z^* \cdot \mathrm{SE} = 135.94 \pm 1.96 \cdot 9.9/\sqrt{24} = 135.94 \pm 3.96 = (131.98, 139.90).$$

Minitab command and output for this calculation:

```
Name C3 'Weight(pounds)'
Let 'Weight(pounds)' = 'Weight(kg)' * 2.2
OneZ 'Weight(pounds)';
 Sigma 9.9;
 Confidence 95.0;
 Alternative 0.
```

One-Sample Z: Weight(pounds)

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
24	135.94	10.58	2.02	(131.98, 139.90)

μ: mean of Weight(pounds)
Known standard deviation = 9.9