Supplementary exercise 6.33 of IPS7e

Time spent studying statistics in an introductory stats class. Let $X_1, \ldots, X_{25}$ denote the times for the 25 students.

We assume that the observations are i.i.d. (independent and identically distributed) with mean μ and standard deviation σ. We also assume (quite unrealistically) that σ is known: $\sigma = 35$ min. The average studying time is given as: $\hat{\mu} = \bar{X} = 80$ min.

(a) With a confidence level of 95%, our z^*-value is 1.96:

$$95\%\text{ CI for }\mu\ :\ \bar{X} \pm z^* \cdot \sigma/\sqrt{n} = 80 \pm 1.96 \cdot 35/\sqrt{25} = 80 \pm 13.72 = (66.3, 93.7).$$

(b) No. The confidence interval is for the population mean, not for values of individual students. We can be 95% confident that the population mean study time is between 66.3 and 93.7. The population includes all students taking the class in question, and perhaps including also students taking similar classes elsewhere with the same textbook and schedule.

The individual study times are much more variable than the mean. If we, for illustration, assumed the study times to follow a normal distribution $\mathrm{N}(80, 35)$, then an estimated 95% range in this distribution would be, using for example the "68−95−99.7%" rule:

$$80 \pm 1.96 \cdot 35 = 80 \pm 68.6 = (11.4, 148.6).$$

This is very different from the confidence interval. We will later encounter the term "prediction interval" for such an interval, referring to the fact that it is the information we have about a (new) value from the distribution, based on the data (of which we only utilized that $\bar{X} = 80$). The terminology is less important here, but it is important to distinguish an interval for individual observations (the prediction interval) from an interval for the mean parameter (the confidence interval). Note that when the sample size (n) increases, the confidence interval shrinks but the prediction interval does not change.

Minitab listings for the confidence interval in (a) and a Probability Distribution Plot for the (assumed) distribution interval in (b):

Descriptive Statistics

N	Mean	SE Mean	95% CI for μ
25	80.00	7.00	(66.28, 93.72)

μ: population mean of Sample
Known standard deviation = 35