Epidemiology/Biostats VHM 812/802 Course
Winter 2015, Atlantic Veterinary College, PEI
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Lecture 3a: Model building II

Index **Page**

- Datasets
  - daisy2red.dta
  - coleman.dta

Model building strategies

- parsimony vs fit
- goals
  - prediction
  - estimates of effects
- **steps**
  - specify maximum model
  - address issues of missing values
  - functional form (linearity) of continuous predictors
  - criterion for selection
  - selection strategy
  - analysis
  - evaluate reliability
  - present results

Specifying the maximum model

- outcome of interest
- key predictors
- important confounders
- other variables of interest
  - lots / few
- causal model **DO NOT FORGET**
  - identify confounders
  - identify intervening variables
  - identify exposure-independent variables

Reducing the number of predictors

- 10 obs. per predictor
- screening – descriptive statistics
  - few missing values
  - substantial variability
  - small categories
- correlation
  - pairwise
- unconditional associations
  - liberal P-value
- multivariate analysis (eg princ. comp.)
- missing values
  - complete case analysis
    - any missing value – entire observation ignored
  - more on missing data – discussed later

Functional form of continuous predictors (linearity)

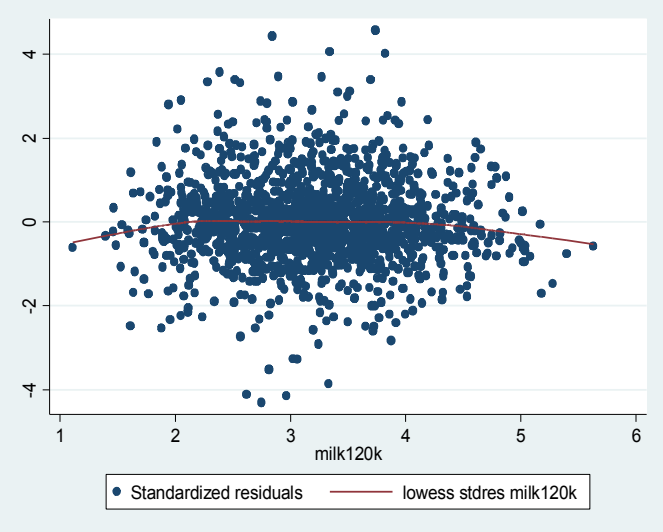
- detecting non-linearity – in final model
  - plot residuals vs fitted values
    - simultaneous evaluation of all predictors
  - plot of residuals vs predictor
- detecting non-linearity – before / during model building
  - scatterplot of outcome vs predictor
    - smoothing functions
- smoothed scatter-plots
  - best fitting line through a mass of data (not confined to any specific shape)
  - local-influence property
    - position of line only affected by "neighbours"
    - # of neighbours determined by bandwidth (width of the neighbourhood)
    - adjust bandwidth to control degree of smoothing
  - different types of smoother
    - lowess – commonly used
  - cautions
    - potential to mask important local effects
    - behave poorly at ends

Detecting and correcting non-linearity

- categorization of predictor (L2a)
  - indicator dummy variable
  - hierarchical dummy variable
- compare categorical and linear variables
  - eg. parity (L1b)
- transformation of X
  - Box-cox analysis
  - polynomial functions of X
    - quadratic, cubic, etc
    - fractional polynomials
    - orthogonal polynomials
- Box-Cox analysis (for Xs)
  - same idea as for the outcome (L1a)
  - regress ln_cf milk120k
    - transformed outcome improved model assumptions

```
. reg ln_cf milk120k - R2= 0.0012 and NS

------------------------------------------------------------------------------
      ln_cf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
------------+----------------------------------------------------------------
   milk120k |   .0134226   .0100303     1.34   0.181    -.0062521    .0330973
      _cons |   4.182258   .0330285   126.63   0.000     4.117471    4.247044
------------------------------------------------------------------------------

. boxcox ln_cf milk120k , model(rhs)
------------------------------------------------------------------------------
      ln_cf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
------------+----------------------------------------------------------------
    /lambda |  -3.647194   1.174882    -3.10   0.002     -5.94992   -1.344468
------------------------------------------------------------------------------

-------------------------------------------------------------
   Test          Restricted     LR statistic      P-value
    H0:       log likelihood       chi2       Prob > chi2
-------------------------------------------------------------
lambda = -1      -181.5923         3.45           0.063
lambda =  0     -182.42186         5.10           0.024
lambda =  1     -183.07949         6.42           0.011
-------------------------------------------------------------
```

- lambda (power parameter) = -3.65
- also inverse transformation (lambda = -1)



- try with quadratic term

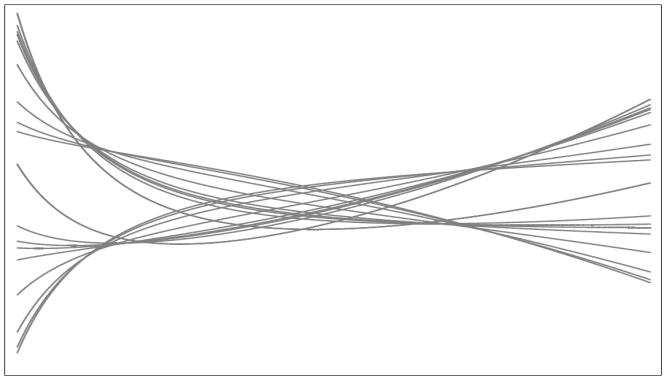
Polynomial functions of X

- quadratic (details L1b and L2b)

- fractional polynomials
  - extension polynomial regression
    - allow log, non-integer powers, repeated powers
  - select terms (usually one or two) of the form x^p
  - where "p" is form the set -2, -1, -0.5, 0, 0.5, 1, 2, 3
    - p=0 is taken to be ln(X)
    - eg $\beta_1 X^{-1}+\beta_2 X^2=\beta_1(1/X)+\beta_2(X^2)$
  - combination selected based on best fit (smallest log likelihood)

- usually 2 power terms (2 degree) can fit most shapes
  - 2-degree FP: x(-2,2) … $x^{(-2)}$ + $x^{(2)}$

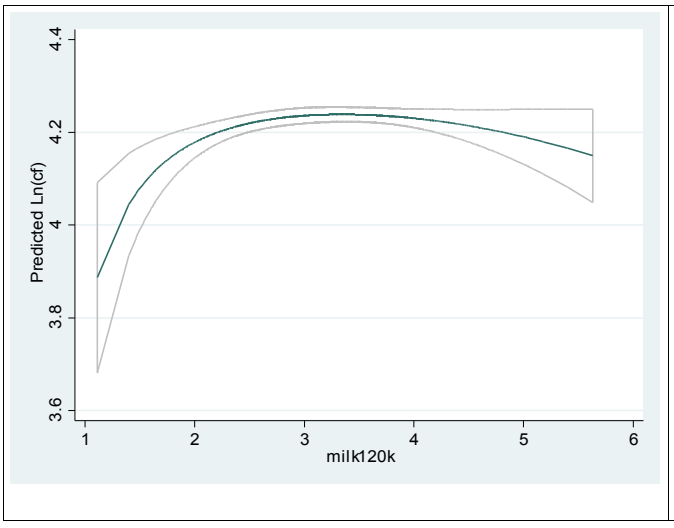
- this graph shows some of the possibilities from a 2-degree FP

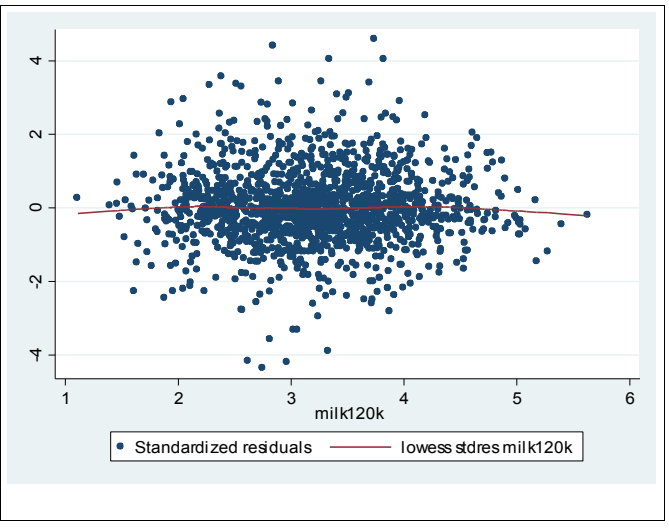


Example – (log) calving to first service and milk120

```
. fp <milk120k>,  scale center replace: reg ln_cf <milk120k>
(fitting 44 models)
(....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%)

Fractional polynomial comparisons:
------------------------------------------------------------------------------
    milk120k |   df    Deviance  Res. s.d.   Dev. dif.   P(*)   Powers
-------------+----------------------------------------------------------------
     omitted |    0    367.951      0.273     10.533    0.033
      linear |    1    366.159      0.273      8.741    0.033   1
       m = 1 |    2    361.396      0.273      3.978    0.138   -2
       m = 2 |    4    357.418      0.272      0.000       --   -2 3
------------------------------------------------------------------------------
(*) P = sig. level of model with m = 2 based on F with 1520 denominator dof.

      Source |       SS       df       MS              Number of obs =    1525
-------------+------------------------------           F(  2,  1522) =    5.27
       Model |  .782289941     2  .391144971           Prob > F      =  0.0052
    Residual |  112.870944  1522  .074159622           R-squared     =  0.0069
-------------+------------------------------           Adj R-squared =  0.0056
       Total |  113.653234  1524  .074575613           Root MSE      =  .27232

------------------------------------------------------------------------------
       ln_cf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  milk120k_1 |  -.5301266   .1654544    -3.20   0.001    -.8546694   -.2055839
  milk120k_2 |  -.0008516   .0004271    -1.99   0.046    -.0016894   -.0000138
       _cons |   4.238434    .008295   510.96   0.000     4.222163    4.254704
------------------------------------------------------------------------------
```

.





- orthogonal polynomials
  - similar to fractional polynomial
  - different parametrization
    - generates polynomials that have zero correlation with powers 1, 2, 3, etc.

Interactions

- 2 way
  - all possible
  - significant main effects
  - significant unconditional assoc.
  - biologically meaningful
  - with key predictor of interest
- 3 way
  - rarely

Selection criteria

- Non-statistical
  - predictor of interest
  - known confounder
  - evidence of being a confounder
  - component of an interaction term
- statistical - nested models
  - F-test for the predictor
  - Wald test or Likelihood ratio test (LRT)
  - always use these tests if appropriate

- other procedures – statistical – non-nested models
  - adjusted R^2 = $1-\frac{\text{MSE}}{\text{MST}}$
    - R^2 adjusted for the # predictors
    - linear regression models only
  - Mallow's Cp
    - linear regression only
    - $C_{pc}=\frac{\text{RSS}}{\sigma^2}+2\text{k}-n$

- usually a positive value – might be negative if many predictors
- lowest Cp = best

- information criteria
  - AIC (Akaike's information criterion)
  - BIC (Bayesian information criterion)

Selection strategies

- all possible / best subset
  - look at all possible combinations of predictors
  - select best model based on some criterion (such as adjusted R^2 or Mallow's CP)
  - best subset – computer finds "best" model with 1,2,3, etc predictors
- Example: coleman.dta – 20 schools in USA
  - outcome: test score 6th graders
  - predictors: staff salary, ses, educ mothers, etc (see 1bl)

- Stata
  - ad-on command: "vselect"

. vselect y_test_scr x1_staff_sal x2_father_job x3_ses x4_test_teach x5_edu_mother, best

```
Response :              y_test_scr
Fixed Predictors :
Selected Predictors:   x3_ses x4_test_teach x1_staff_sal
x5_edu_mother                                x2_father_job

Actual Regressions   5
Possible Regressions 32

 Optimal Models Highlighted:
   # Preds     R2ADJ         C       AIC      AICC       BIC
         1  .8518292  4.974883  90.89429  149.1518  92.88576
         2  .8740953  2.832759  88.49432  147.9185  91.48152
         3  .8820943  2.836089  87.96903  149.0123  91.95196
         4  .8756399  4.670221  89.74417  152.9633  94.72284
         5  .8728444         6  90.80893  156.8998  96.78332
Selected Predictors

1  :  x3_ses
2  :  x3_ses x4_test_teach
3  :  x3_ses x4_test_teach x1_staff_sal
4  :  x3_ses x4_test_teach x1_staff_sal x5_edu_mother
5  :  x3_ses x4_test_teach x1_staff_sal x5_edu_mother x2_father_job
```

Stepwise estimation

- stepwise command in Stata
- old syntax

- forward selection
  - start with a null model
  - adds terms based on statistical significance (one at a time, always choosing the most significant predictor not yet in the model)
  - stop when no more terms are significant when added

```
. stepwise, pe(0.1): reg y_test_scr x1_staff_sal x2_father_job x3_ses x4_test_teach
x5_edu_mother
                     begin with empty model
p = 0.0000 <  0.1000  adding  x3_ses
p = 0.0566 <  0.1000  adding  x4_test_teach

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  2,    17) =   66.95
       Model |  570.497872     2  285.248936            Prob > F      =  0.0000
    Residual |  72.4264222    17  4.26037777            R-squared     =  0.8873
-------------+------------------------------           Adj R-squared =  0.8741
       Total |  642.924294    19  33.8381207            Root MSE      =  2.0641

------------------------------------------------------------------------------
   y_test_scr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
       x3_ses |   .5415607   .0500441    10.82   0.000     .4359768    .6471445
x4_test_teach |   .7498915   .3666402     2.05   0.057    -.0236517    1.523435
        _cons |    14.5827   9.175409     1.59   0.130    -4.775723    33.94112
------------------------------------------------------------------------------
```

- backward elimination
  - starts with a full model
  - eliminates terms that are not significant (one at a time, starting with the "least significant")
  - stop once all terms remaining in the model are significant

```
. stepwise, pr(0.1): reg y_test_scr x1_staff_sal x2_father_job x3_ses x4_test_teach
x5_edu_mother
                      begin with full model
p = 0.4267 >= 0.1000  removing x2_father_job
p = 0.6863 >= 0.1000  removing x5_edu_mother
p = 0.1616 >= 0.1000  removing x1_staff_sal

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  2,    17) =   66.95
       Model |  570.497872     2  285.248936           Prob > F      =  0.0000
    Residual |  72.4264222    17  4.26037777           R-squared     =  0.8873
-------------+------------------------------           Adj R-squared =  0.8741
       Total |  642.924294    19  33.8381207           Root MSE      =  2.0641

------------------------------------------------------------------------------
   y_test_scr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
       x3_ses |   .5415607   .0500441    10.82   0.000     .4359768    .6471445
x4_test_teach |   .7498915   .3666402     2.05   0.057    -.0236517    1.523435
        _cons |    14.5827   9.175409     1.59   0.130    -4.775723    33.94112
------------------------------------------------------------------------------
```

- stepwise
  - combines forward and backward
  - generally preferred approach is stepwise backward
    - starts with a full model and works backward using a stepwise approach

```
. stepwise, pe(0.1) pr(0.11): reg y_test_scr x1_staff_sal x2_father_job x3_ses
x4_test_teach x5_edu_mother
                     begin with full model
p = 0.4267 >= 0.1100  removing x2_father_job
p = 0.6863 >= 0.1100  removing x5_edu_mother
p = 0.1616 >= 0.1100  removing x1_staff_sal

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  2,    17) =   66.95
       Model |  570.497872     2  285.248936           Prob > F      =  0.0000
    Residual |  72.4264222    17  4.26037777           R-squared     =  0.8873
-------------+------------------------------           Adj R-squared =  0.8741
       Total |  642.924294    19  33.8381207           Root MSE      =  2.0641

------------------------------------------------------------------------------
   y_test_scr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
       x3_ses |   .5415607   .0500441    10.82   0.000     .4359768    .6471445
x4_test_teach |   .7498915   .3666402     2.05   0.057    -.0236517    1.523435
        _cons |    14.5827   9.175409     1.59   0.130    -4.775723    33.94112
------------------------------------------------------------------------------
```

- daisy2red dataset
  - reg wpc_sqrt parity1 aut_clv herd_size hs_sq dyst twin twdy rp vag_disch rpvd

```
*stepwise backward
stepwise, pe(0.05) pr(0.051) lockterm1: reg sqrt_wpc  parity1 aut_clv (hs_ct hs_sq)
      (dyst twin twdy) (rp vag_disch rpvd)
estimates store sw_1

stepwise, pe(0.05) pr(0.051) lockterm1: reg sqrt_wpc  parity1 aut_clv (hs_ct hs_sq)
      dyst twin twdy rp vag_disch rpvd
estimates store sw_2
estimates table sw_1 sw_2
```

```
. estimates table sw_1 sw_2
----------------------------------------
    Variable |    sw_1         sw_2
-------------+--------------------------
     parity1 |  .05586593    .04388077
     aut_clv | -.51283075   -.52440137
       hs_ct | -.02346682   -.02161068
       hs_sq |   .0000713    .00006694
        dyst |  .62771805
        twin |  1.6982381    1.4787911
        twdy | -2.7727951
          rp |  .39042354
   vag_disch | -.04220258
        rpvd |  1.4760578    1.8317287
       _cons |  3.0230207    3.4048174
----------------------------------------
```

Cautions with automated selection procedures

- don't let your computer decide what variables go into your model
- R^2 too high
- F-tests too large
- severe problems if collinearity
- ignores non-statistical considerations
  - exposures, confounders and intervening var.
- you need to take care of
  - dummy variables
  - interaction terms
  - missing data
- useful when faced with large number of predictor variables
  - help to identify predictors that potentially are statistically significant associated with the outcome
- perform residual and influential analysis of selected models

Evaluate reliability

- validity
  - regression diagnostics
- reliability
  - ability to predict future observations
  - split sample analysis
    - split data into two parts (eg. 50:50)
    - build model using one part (1st model), generate predictions and compare them with observed values for the other part (2nd model)
    - cross-validation correlation
      - corr. predicted and obs. values 2nd mod.
    - shrinkage on cross-validation
      - difference R^2 from the 1st model and the square of the cross-validation correlation
      - guideline: <0.1 is Ok
        - but will depend on the scale

- leave-one-out analysis
  - fit the model using all data except one observation
  - generate prediction for the left-out obs.
  - compare the **P**redicted **R**esidual **S**um of **S**quares (PRESS) from predicted points to prediction error from full model
  - PRESS = sum{residual$_i$ / (1-leverage$_i$)^2}
  - compare R^2 (prediction) vs R^2 (full model)
    - eg. R^2(prediction) = (1-PRESS)/Total Error

- Example: daisy2red dataset
  - split-sample analysis

```
. gen rand=uniform()

. reg  wpc_sqrt c.hs_ct##c.hs_ct parity1 aut_calv twin i.dyst##vag_disch if
rand<0.6, vsquish

      Source |       SS       df       MS              Number of obs =     920
-------------+------------------------------           F(  8,   911) =   12.41
       Model |  807.033629     8  100.879204           Prob > F      =  0.0000
    Residual |  7405.23534   911  8.12868863           R-squared     =  0.0983
-------------+------------------------------           Adj R-squared =  0.0904
       Total |  8212.26897   919  8.93609246           Root MSE      =  2.8511

------------------------------------------------------------------------------
       wpc_sqrt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
----------------+-------------------------------------------------------------
          hs_ct |   .0134746   .0016229     8.30   0.000     .0102897    .0166596
c.hs_ct#c.hs_ct |   .0000853   .0000237     3.60   0.000     .0000389    .0001318
        parity1 |    .064454   .0638652     1.01   0.313    -.0608859     .189794
       aut_calv |  -.5758069   .1899446    -3.03   0.003    -.9485868   -.2030269
           twin |   1.597395   .8017033     1.99   0.047     .0239953    3.170795
           dyst |
            yes |   1.512701   .4427911     3.42   0.001     .6436914     2.38171
      vag_disch |
            yes |   1.358874   .4449721     3.05   0.002     .4855847    2.232164
 dyst#vag_disch |
      yes#yes |  -4.092153    1.13334    -3.61   0.000    -6.316413   -1.867892
          _cons |   7.454233   .1914211    38.94   0.000     7.078556    7.829911
------------------------------------------------------------------------------
```

```
. predict pv
(option xb assumed; fitted values)

. *cross validation correlation
. corr pv wpc_sqrt if rand>=0.6
(obs=654)

             |       pv wpc_sqrt
-------------+------------------
          pv |   1.0000
    wpc_sqrt |   0.2409   1.0000


. *shrinkage on cross-validation
. scalar cv_sq = r(rho)^2   /* this computes r2 - r(rho) is a "saved result" from
-corr-  */

. di "R2 first model= " r2_1 "  and CV2= " cv_sq
R2 first model= .0982717  and CV2= .05804935

. di "shrinkage on cross-validation = " r2_1-cv_sq
shrinkage on cross-validation = .04022235
```

leave-one-out analysis (PRESS analysis)

```
. predict res, res

. predict lev ,lev

. gen eq1=(res/(1-lev))^2

. summ eq1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         eq1 |      1574    7.851331     10.9765   1.85e-07   83.92817

. di "PRESS =" r(sum) //error sum square from predicted points
PRESS =12357.995

. di "ESS full="e(rss) //residuals sum square from full model
ESS full=12203.975


. di "R2(pred) = "1-(r(sum)/ (`e(mss)' + `e(rss)'))
R2(pred) = .07126506

. di "R2 full model =" e(r2)
R2 full model =.08284007
```

Presenting the results

- standardized coefficients
  - scales all coefficients so that they represent a change of 1 SD.

$$\beta^{*}=\beta(\sigma_x/\sigma_y)$$

    - option "beta"
      - eg. regress sqrt_wpc parity twin, beta
  - compare the relative magnitude of several predictors
    - use with caution to compare coeff. from diff. Studies

```
. reg  wpc_sqrt c.hs_ct##c.hs_ct parity1 aut_calv twin dyst##vag_disch , beta
vsquish

------------------------------------------------------------------------------
       wpc_sqrt |      Coef.   Std. Err.      t    P>|t|                    Beta
----------------+-------------------------------------------------------------
          hs_ct |   .0124288   .0012117    10.26   0.000                .2650174
c.hs_ct#c.hs_ct |   .0000717   .0000174     4.13   0.000                .1063269
        parity1 |   .0624408     .04795     1.30   0.193                .0320708
       aut_calv |  -.5313957   .1419088    -3.74   0.000               -.0912394
           twin |   1.484421   .5487805     2.70   0.007                .0662909
           dyst |
           yes  |   .8159669   .3267812     2.50   0.013                .0665023
      vag_disch |
           yes  |   .9922199   .3503534     2.83   0.005                .0758348
 dyst#vag_disch |
       yes#yes  |  -2.257598   .8832373    -2.56   0.011               -.0729025
          _cons |   7.529392   .1445102    52.10   0.000                       .
------------------------------------------------------------------------------
```

- interquartile ranges (IQR)
  - how big is the change in the outcome if the predictor changes through out the IQR
    - IQR = diff. between 75th and 25th percentile
      - parity IQR = 3, coef. = 0.06
      - effect = 0.06*3= 0.18 units
    - avoid impact of outlying observations

- predictors eliminated from the model
  - don't ignore predictors just because they have been eliminated from the model
    - not statistically sig. ≠ no effect
    - unconditional associations?
    - force back in to the final model?
- scale of results
  - dealing with transformed data
  - compute some expected effects of key predictors on the original scale at varous levels of the other factors

A Structured Approach to Data Analysis (VER 30)

- "jump to the finish"
  - start over
- data collections sheets
- data coding
- data entry
- keeping track of files
- keeping track of variables
- analyses
  - data editing
  - data verification
  - data processing - outcome variable
  - data processing - predictor variables
  - data processing - multilevel
  - unconditional associations
- keeping track of analyses