Epidemiology/Biostats VHM 812/802 Course
Winter 2016, Atlantic Veterinary College, PEI
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Lecture 5a: Logistic regression diagnostics

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Friday

- logistic regression exercise 16.3
- conditional logistic regression (and exact logistic regression) or
- last chance to work on exercises!!

Dataset= nocardia.dta

- all the examples based on VER Ex. 16.5 (model with dcpct3, dneo, dclox and dneo*dclox)

Covariate patterns

- covariate pattern
  - unique combination of values predictor variables

Binomial Data $X_1 = 0/1$ $X_2 = 0/1$					
X_1	X_2	Cov. Patter	# pos	n	propn.
0	0	1	6	10	.6
0	1	2	3	20	.15
1	0	3	5	50	.1
1	1	4	4	10	.4

Binary Data X_1 = age (in yrs. to 1 decimal) X_2 = wt in kg. (to 1 decimal)					
X_1	X_2	Cov. Patter	# pos	n	propn.
4.3	527.2	1	0	1	0
3.7	489.6	2	1	1	1
2.1	535.4	3	1	1	1
5.6	501.4	4	0	1	0
		....			

- example

				Residual	
Obs.	Cov. pattern	Disease	Pred. Value	1 per Obs.	1 per Cov. Pat.
1					
2					

Residuals in logistic regression

- one per observation (based on Hilbe, 2009[1])
  - (standard) residual analysis
  - mainly for visual assessment
  - not very useful for assessing the model
  - Stata -glm- command

- one per covariate pattern
  - goodness-of-fit tests
  - residual analysis
    - Pearson residuals (standardized)
    - Deviance residuals (not covered in this course)
  - leverage
  - influential observations
    - delta χ^2 ($\Delta\chi^2$)
    - delta-beta ($\Delta\beta$)
  - Stata command -*logit / logistic* -

1 Hilbe J. Logistic Reg. Models. CRC Press: Boca Raton 2009

Pearson residuals per covariate pattern

- Pearson residual
  - $r_j = \frac{y_j - m_j * p_j}{\sqrt{m_j * p_j * (1 - p_j)}}$
    - y_j = nbr. pos. outcomes in j^{th} covariate pattern
    - m_j = nbr. obs. in the j^{th} covariate pattern
    - p_j = predicted prob. for the j^{th} covariate pattern
  - standardized residuals (same as before)
  - $\sum_{j=1}^{J} (r_j)^2 = \text{Pearson}\,\chi^2\ statistic \sim \chi^2\ with\,(J-k)\ df.$
    - k = number of parameters in the model
  - contribution of each cov. pattern to the χ^2 statistic
  - Stata command – *predict* after *logit/logistic* -
    - eg. logit casecon i.dneo
      - predict pear, residual
      - predict stdpear, rstandard
  - other residuals
    - Deviance
    - Anscombe (more normal distributed - glm)

Goodness of fit tests

- Pearson χ^2 (1 per cov. pattern)
  - χ^2 distributions (J-k) df
    - J = # covariate patterns
    - k = # of parameters in model
  - only if enough number of replications per group (eg cov. patterns)
    - ~ guidelines ~ X^2-statistic
      - more than 1 expected counts in each cell
      - at least 80% expected values > 5 counts
  - indicates the fit of the model
    - Ho = model fits the data

- Example: nocardia

covariate pattern and Case - Control	Nbr herds	Nbr pos	Pre. pr.	Pear. Res.	Pear. Res.^{2}
1					
no	12	1	0.028	1.144	1.308949
yes					
2					
no	2	0	0.102	-0.478	.2283039
yes					
3					
no	8	1	0.182	-0.416	.1731429
yes					
4					
no					
yes	1	1	0.152	2.360	5.569528
5					
no	11	2	0.259	-0.584	.3405482
yes					
6					
no	11	4	0.416	-0.353	.1245677
yes					
7					
no	10	7	0.735	-0.254	.0643712
yes					
8					
no	38	33	0.844	0.416	.1731366
yes					
9					
no	1	0	0.082	-0.298	.0890559
yes					
10					
no	5	1	0.258	-0.295	.0872724
yes					
11					
no	9	4	0.403	0.252	.0634578
yes					

```
. qui summ pear_sq if cnt~=. //command to capture the sum of pear_sq

. di "Pearson X2 = " r(sum) "  Prob > chi2 =" chi2tail(11-6,r(sum))
Pearson X2 = 8.2223332  Prob > chi2 =.14440061
```

- no indication of lack of fit Pearson $\chi 2$ with df=5
- largest contribution is from cov. pattern with no replication (eg cov # 4)

- Pearson χ2 after logit command
  - p-value is meaningful if enough replicates
  - Stata command (after logit)
    - *estat gof*

```
. estat gof

Logistic model for casecont, goodness-of-fit test

       number of observations =       108
 number of covariate patterns =        11
             Pearson chi2(5) =         8.22
                 Prob > chi2 =         0.1444
```

- Hosmer-Lemeshow Test
  - only useful test when there are few replicates per covariate pattern
  - group by:
    - percentiles of estimated probability
    - fixed points of estimated probability
  - compares predicted probabilities to observed probabilities in groups (g) of data
    - χ^2 with g-2 df
    - low power if < 6 groups
  - Stata command (after logit)
    - *estat gof, group(#) table*

- Example

```
. estat gof, g(10) table

Logistic model for casecont, goodness-of-fit test

  (Table collapsed on quantiles of estimated probabilities)
  (There are only 7 distinct quantiles because of ties)
  +--------------------------------------------------------+
  | Group |   Prob | Obs_1 | Exp_1 | Obs_0 | Exp_0 | Total |
  |-------+--------+-------+-------+-------+-------+-------|
  |     1 | 0.0284 |     1 |   0.3 |    11 |  11.7 |    12 |
  |     2 | 0.1817 |     2 |   1.9 |    10 |  10.1 |    12 |
  |     3 | 0.2589 |     3 |   4.1 |    13 |  11.9 |    16 |
  |     4 | 0.4033 |     4 |   3.6 |     5 |   5.4 |     9 |
  |     5 | 0.4161 |     4 |   4.6 |     7 |   6.4 |    11 |
  |-------+--------+-------+-------+-------+-------+-------|
  |     6 | 0.7354 |     7 |   7.4 |     3 |   2.6 |    10 |
  |    10 | 0.8439 |    33 |  32.1 |     5 |   5.9 |    38 |
  +--------------------------------------------------------+

       number of observations =       108
             number of groups =         7
    Hosmer-Lemeshow chi2(5) =         2.16
                Prob > chi2 =         0.8262
```

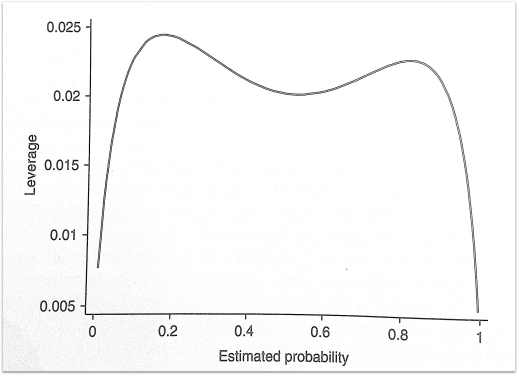
Overdispersion [L11a - L11b]

- assumption y_i ~ binomial distribution
  - mean = $n_i * p_i$
  - variance = $n_i * p_i * (1 - p_i)$
- overdispersion = the data are more dispersed (larger variance) that would be expected
  - apparent overdispersion – wrong model
    - missing important predictors
    - outliers
  - real overdispersion – usually due to clustering
  - too small S.E.

Leverage

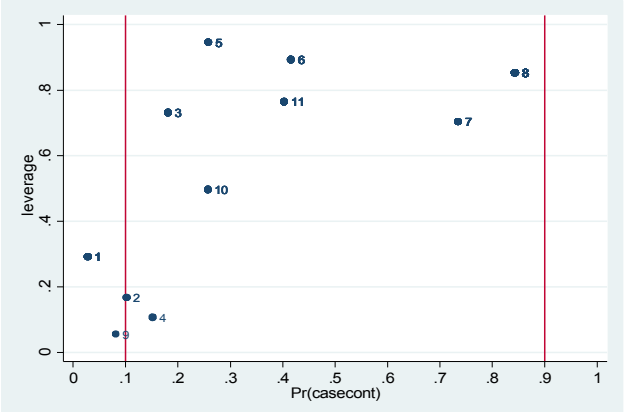
- potential impact of cov. pattern on the model
- extent to which the j^{th} cov. pattern is separated for the others in terms of the explanatory variables
- leverage depends on x's and predicted value

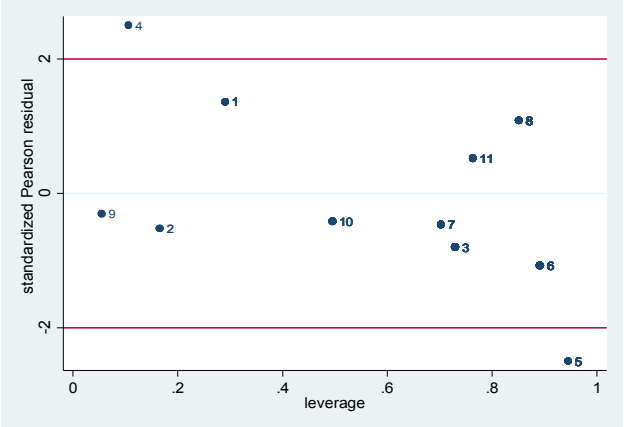
Predicted probabilities	Leverage
0.0 – 0.1	low
0.1 – 0.3	high
0.3 – 0.7	moderate
0.7 – 0.9	high
0.9 – 1.0	low



- If estimated probabilities >0.1 and <0.9 then leverage values can be interpreted as distance
  - look for large leverage values within this range
  - look for points that fall some distance from the rest of the data

- Example



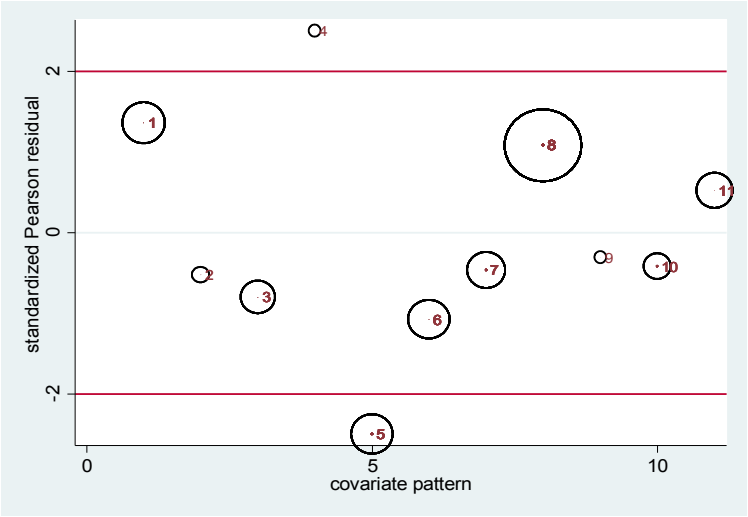


```
. list cov cnt dcpct3 dneo dclox opr pv pear_std lev if  pv>0.1 & pv<0.9 & wcov==1,
noobs

  +-------------------------------------------------------------------------+
  | cov   cnt   dcpct3   dneo   dclox     opr      pv   pear_std     lev |
  |-------------------------------------------------------------------------|
  |   2     2       50     no      no   0.000   0.102     -0.523   0.166 |
  |   3     8      100     no      no   0.125   0.182     -0.801   0.730 |
  |   4     1       50     no     yes   1.000   0.152      2.496   0.106 |
  |   5    11      100     no     yes   0.182   0.259     -2.496   0.945 |
  |   6    11        0    yes      no   0.364   0.416     -1.073   0.892 |
  |-------------------------------------------------------------------------|
  |   7    10       50    yes      no   0.700   0.735     -0.465   0.703 |
  |   8    38      100    yes      no   0.868   0.844      1.080   0.852 |
  |  10     5       50    yes     yes   0.200   0.258     -0.416   0.496 |
  |  11     9      100    yes     yes   0.444   0.403      0.518   0.764 |
  +-------------------------------------------------------------------------+
```

Residual analysis (covariate patterns)

- Pearson residuals
  - standardized residuals
    - 95% between -2 and +2
  - identify large negative and positive residuals
  - characteristics of observations
  - visual assessment
    - some guidelines about outlying obs.
- Example
  - stdz. Pearson residuals (per cov. pattern) vs cov. pattern

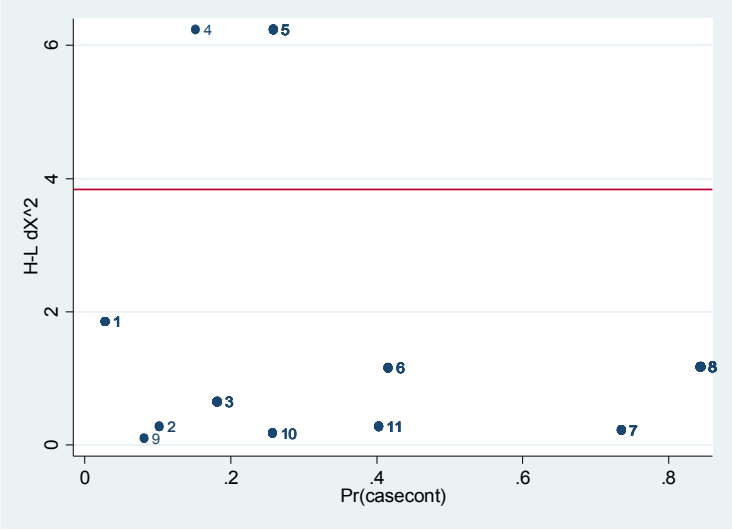


```
. list cov cnt dcpct3 dneo dclox opr pv pear_std if wcov==1 & abs(pear_std)>2,noobs

  +-------------------------------------------------------------------+
  | cov   cnt   dcpct3   dneo   dclox     opr      pv   pear_std |
  |-------------------------------------------------------------------|
  |   4     1       50     no     yes   1.000   0.152      2.496 |
  |   5    11      100     no     yes   0.182   0.259     -2.496 |
  +-------------------------------------------------------------------+
```

Influential statistics

- delta χ^2 ($\Delta\chi^2$)
  - effect of covariate pattern on Pearson χ^2
    - identifies patterns that do not fit well (outliers)
    - plot delta values vs predicted probabilities
    - plot delta values vs leverage
    - delta-values >=3.84 (95th percentiles X^2 distribution with 1df)
- Example - delta χ^2 ($\Delta\chi^2$) vs Pr(Pr)



```
. list cov cnt dcpct3 dneo dclox pv dx2 lev pear if dx2>3.84 & wcov==1, noobs

  +-------------------------------------------------------------------------------+
  | cov   cnt   dcpct3   dneo   dclox         pv        dx2        lev     pear |
  |-------------------------------------------------------------------------------|
  |   5    11      100     no     yes   .2588893   6.232499   .9453593   -0.584 |
  |   4     1       50     no     yes    .152218   6.232499   .1063734    2.360 |
  +-------------------------------------------------------------------------------+
```

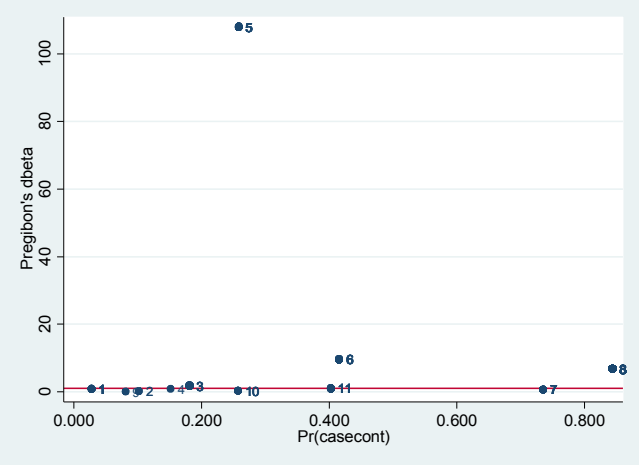
- delta-betas ($\Delta\beta$)
  - ✯ analogous to Cook's distance
  - ✯ measures influence of a cov. pattern on:
    - ➨ overall set of betas (Stata)
    - ➨ individual betas (SAS)
  - ✯ depends on leverages and # of observations (m_j) on the covariate pattern variable
    - ➨ Hosmer & Lemeshow[2] suggest that values >1 might be influential
- Leverage, $\Delta\chi^2$ and $\Delta\beta$
  - ✯ values will depend on the predicted probabilities (similar to leverage)[2]

Predicted probabilities	Leverage	$\Delta\chi^2$	$\Delta\beta$
0.0 – 0.1	small	large or small	small
0.1 – 0.3	large	moderate	large
0.3 – 0.7	moderate	moderate	moderate
0.7 – 0.9	large	moderate	large
0.9 – 1.0	small	large or small	small

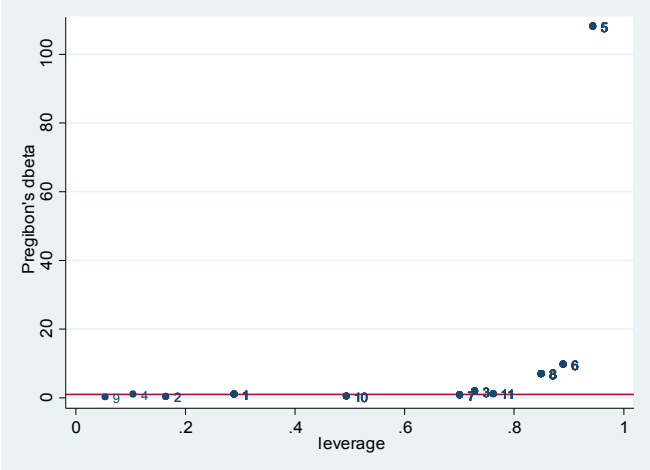
2 Hosmer and Lemeshow. Applied Log. Reg. 2nd Edition. pg-174-176

- Plots

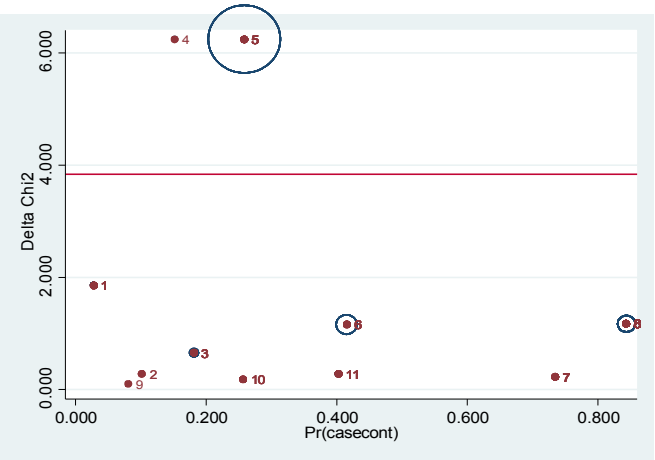
$\Delta\beta$ vs pred. prob.



$\Delta\beta$ vs leverage values



- $\Delta\chi^2$ vs predicted probabilities with size proportional to $\Delta\beta$



- influential observations

```
. l cov cnt dcpct dneo dclox opr pv lev dx2 db if db > abs(1) & wcov==1, noobs

  +-------------------------------------------------------------------------------+
  | cov   cnt   dcpct   dneo   dclox     opr      pv     lev     dx2          db |
  |-------------------------------------------------------------------------------|
  |   3     8     100     no      no   0.125   0.182   0.730   0.642       1.739 |
  |   8    38     100    yes      no   0.868   0.844   0.852   1.167       6.693 |
  |   6    11       5    yes      no   0.364   0.416   0.892   1.152       9.504 |
  |   5    11     100     no     yes   0.182   0.259   0.945   6.232     107.831 |
  +-------------------------------------------------------------------------------+
```

- delta-betas extreme for cov. 5 (same as VER), 6 (not in VER) and 8 (noted in VER to be due to a large group size)

Dealing with influential observations

- identify points with large residuals or large leverage values
- evaluate their covariate patterns – why are they outliers?
- delete from model and re-fit the model
  - does it change very much?

```
. estimates table final wocov5 wocov6 wocov8 , b(%5.3f)  stats(N) star( .05
.01 .001)

-------------------------------------------------------------------
    Variable |   final        wocov5       wocov6       wocov8
-------------+-----------------------------------------------------
        dneo |
         yes |   3.192***     3.248***     3.639***     2.518*
             |
       dclox |
         yes |   0.453       17.322        0.808        0.705
             |
  dneo#dclox |
    yes#yes  |  -2.533*     -19.403       -3.018*      -2.053
             |
      dcpct3 |
          50 |   1.361        1.087        0.120        1.173
         100 |   2.027**      2.132**      0.813        1.168
             |
       _cons |  -3.531***    -3.581***    -2.672*      -2.972**
-------------+-----------------------------------------------------
           N |     108           97           97           70
-------------------------------------------------------------------
                            legend: * p<.05; ** p<.01; *** p<.001
```

- cov pattern 5 – only cov. with dneo=0 and dclox=1 case and controls – part of the interaction
- cov pattern 6 – dneo=1 and dclox=0 with 0 dcpct
- cov pattern 8 – largest cov. pattern
- cov pattern 4 – largest contribution to the deviance and Pearson X2 – however no influence (delta-beta = 0.74)

Summary logistic regression diagnostics

- covariate pattern residuals
  - goodness-of-fit tests
    - inadequacies in the modelling of the predictors in the model
      - e.g non-linearity or missing interactions
    - can't detect missing predictors or clustering
  - outlying observations (cov. patterns)
- diagnostics
  - consequences of the current model
    - eg. few replicates?
    - increase the nbr of obs. per cov. pattern (or reduce the nbr. of cov. patterns)
      - grouping (biological, frequency categories)
      - remove continunous predictors
        - eg. those that are borderline sig.
    - however.... you will be working with a different model – not your final model!
  - identify high influence cov. patterns (for instance $\Delta\beta$)on the parameter estimates

Predictive ability of a logistic model

$$\ln\left(\frac{p}{1\text{-}p}\right)=\beta_0+\beta_1*X_1=X\beta \qquad p=\frac{1}{1+e^{-(\beta X)}}$$

- note: not a true probability if derived from a case-control study [see l3b]

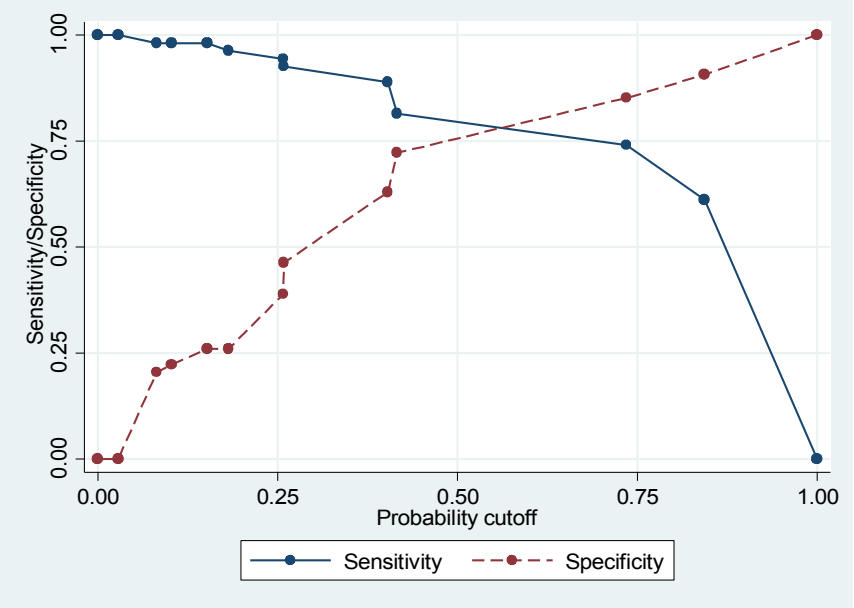
Sensitivity and Specificity

- predict D+ if p >= 0.5
  - choose other cutpoint

Classified (predicted status)	true D+	true D-	Total
T+ = p(D+) >= 0.5	40	8	48
T- = p(D+) < 0.5	14	46	60
Total	54	54	108

- sensitivity (Se) =
- specificity (Sp) =
- positive predictive value (PPV) =
- negative predictive value (NPV) =
- overall correctly classified =

- two-graph ROC (Se-Sp plot)
  - ★ effect of changing the cutpoint on Se and Sp.




```
. egen pv_cat=cut(pv), at(0(.05)1)
. roctab casecont pv_cat, graph sum detail scheme(s1mono)

Detailed report of sensitivity and specificity
```

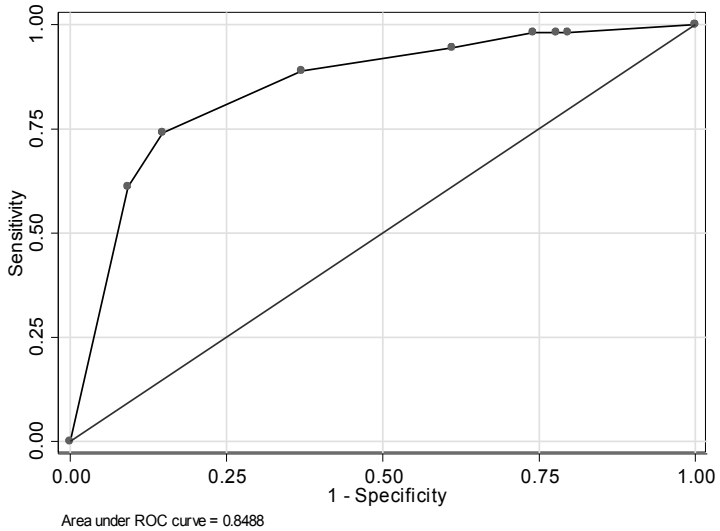
Cutpoint	Sensitivity	Specificity	Correctly Classified	LR+	LR-
(>= 0)	100.00%	0.00%	50.00%	1.0000	
(>= .05)	98.15%	20.37%	59.26%	1.2326	0.0909
(>= .1)	98.15%	22.22%	60.19%	1.2619	0.0833
(>= .15)	98.15%	25.93%	62.04%	1.3250	0.0714
(>= .25)	94.44%	38.89%	66.67%	1.5455	0.1429
(>= .4)	88.89%	62.96%	75.93%	2.4000	0.1765
(>= .7)	74.07%	85.19%	79.63%	5.0000	0.3043
(>= .8)	61.11%	90.74%	75.93%	6.6000	0.4286
(> .8)	0.00%	100.00%	50.00%		1.0000

- ROC curve
  - Se vs 1-Sp
  - assess discriminatory power of the model
    - eg. probability that cases (or controls) are correctly classified by the model (at a given cutpoint)
  - quantify Area Under the Curve (AUC)
  - AUC interpretation[1]

AUC	Interpretation
0.5	no discrimination (better flip a coin!)
0.5 – 0.7	good
0.7 - 0.8	very good
>0.9	excellent

- AUC = final model AUC= 0.85 (see commands do-file)

Obs	ROC Area	Std. Err.	-Asymptotic Normal-- [95% Conf.	Interval]
108	0.8488	0.0370	0.77621	0.92132



1 Adapted from Hosmer Lemeshow. Aplied logistic regression (pg162)

- Concordant pairs (Minitab/SAS)
  - total # of pairs of obs. with different outcomes
    - total pairs = n1 * n0 (eg 54 cases * 54 controls) = 2916
    - concordant pairs = $\hat{p}_1 > \hat{p}_0$
    - discordant pairs = $\hat{p}_1 < \hat{p}_0$
    - tied pairs = $\hat{p}_1 = \hat{p}_0$
  - Area Under the Curve (AUC)
    - AUC = $\dfrac{\text{concordant pairs} + 0.5 * \text{Nbr. ties}}{\text{total pairs}}$
      - concordant pairs = 2322
      - tied pairs = 291
      - total pairs = 2916
      - AUC = (2322+0.5*291)/2916 = 0.8462