

## Lecture 2b: Linear Regression Diagnostics

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- Exercises

- ★ Tuesday Jan 22 - linear regression exercise 3

- Wednesday Quiz 1

## Example: WPC model (daisy2red.dta)

- VER example 14.12

- ★ outcome: wpc (interval from waiting period to conception)

- ★ predictors: parity, twin, dyst, rp, vag\_disch, herd\_size and herd\_size2, calving\_date (in months)

- ➔ also interactions: rp\*vag\_disch

- Final (candidate) model

```
. reg wpc hs100_ctr hs100_ctr_sq parityl i.aut_calv i.twin i.dyst i.rp##i.vag_disch
```

Source	SS	df	MS	Number of obs	=	1,574
Model	296062.694	9	32895.8549	F(9, 1564)	=	13.22
Residual	3892027.86	1,564	2488.50886	Prob > F	=	0.0000
				R-squared	=	0.0707
				Adj R-squared	=	0.0653
Total	4188090.56	1,573	2662.48605	Root MSE	=	49.885

wpc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
hs100_ctr	19.85708	2.163397	9.18	0.000	15.61361 24.10054
hs100_ctr_sq	11.13827	3.111145	3.58	0.000	5.035817 17.24073
parityl	1.13721	.8583103	1.32	0.185	-.5463501 2.82077
aut_calv					
yes	-8.263839	2.537751	-3.26	0.001	-13.24159 -3.286086
twin					
yes	20.68314	9.845165	2.10	0.036	1.37203 39.99425
dyst					
yes	11.70041	5.462576	2.14	0.032	.985666 22.41516
rp					
yes	5.98687	4.811976	1.24	0.214	-3.451734 15.42547
vag_disch					
yes	1.228196	7.161395	0.17	0.864	-12.81875 15.27514
rp#vag_disch					
yes#yes	22.85194	12.51605	1.83	0.068	-1.698056 47.40194
_cons	64.33029	2.634114	24.42	0.000	59.16352 69.49705

# Evaluating major assumptions

## ● Homoscedasticity

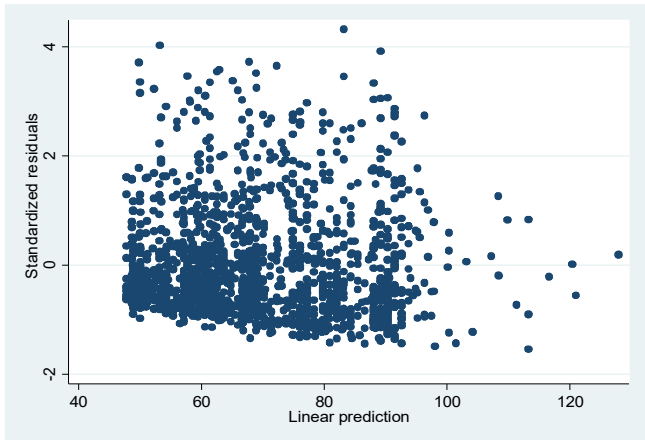
```
. hettest

Breusch-Pagan / Cook-Weisberg test
for heteroskedasticity
    Ho: Constant variance
    Variables: fitted values of wpc

    chi2(1)      =    20.58
    Prob > chi2  =    0.0000

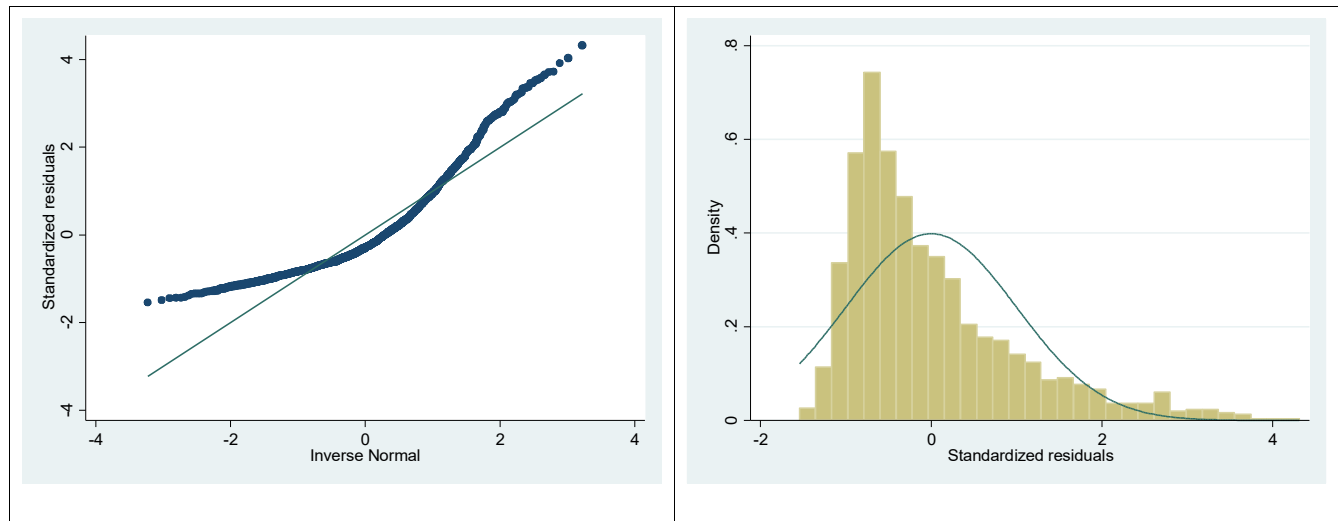
. imtest

Cameron & Trivedi's decomposition of IM-test
```



Source	chi2	df	p
Heteroskedasticity	74.11	44	0.0030
Skewness	143.84	9	0.0000
Kurtosis	33.27	1	0.0000
Total	251.22	54	0.0000

## ● Normality



```
. swilk stdres

Shapiro-Wilk W test for normal data
```

Variable	Obs	W	V	z	Prob>z
stdres	1574	0.87871	115.660	11.977	0.00000

## Transformation of Y (L1a - p13-17)

- Box-cox
- Interpretation

## Re-assessing assumptions

- Log-transformed model

```
xi:boxcox wpc hs100_ctr hs100_ctr_sq parity1 i.aut_calv i.twin i.dyst  
i.rp*i.vag_disch
```

...output omitted...

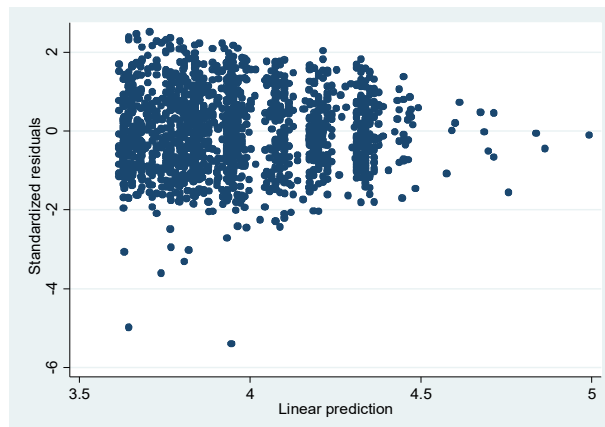
```
Log likelihood = -7960.5368
```

Number of obs	=	1574
LR chi2(8)	=	148.83
Prob > chi2	=	0.000

wpc	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
/theta	.1099104	.0271003	4.06	0.000	.0567947 .1630261

★ value of  $\lambda$  close to 0  $\rightarrow$  log transformation

- Homoscedasticity (L1a - p12)

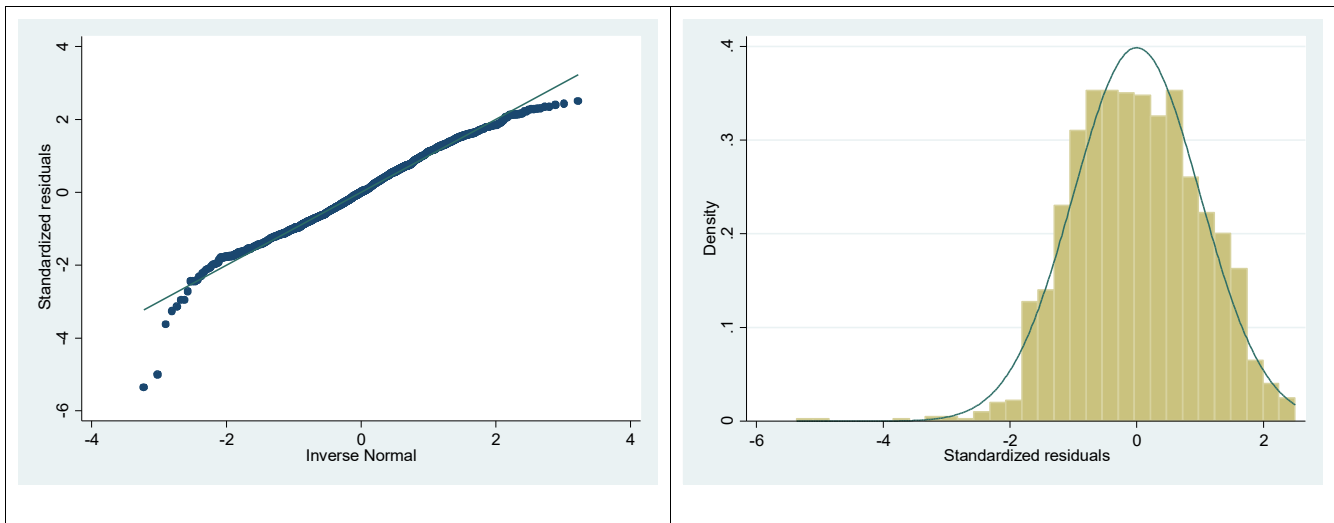


★ tests provide contradicting results

$\rightarrow$  imtest – constant variance

$\rightarrow$  however BPCW test highly significant

● Normality (L1a - p11)

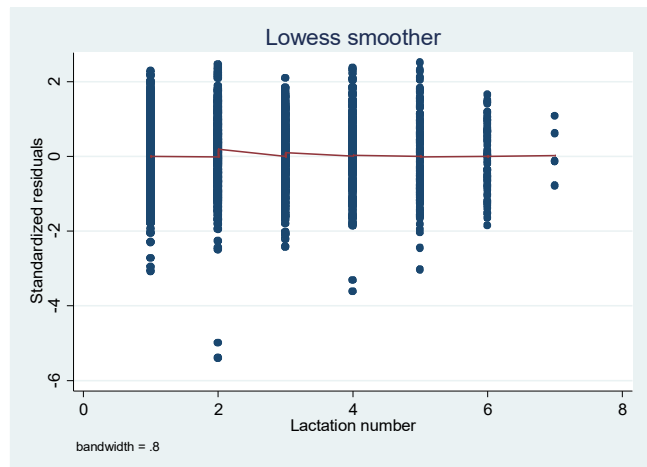
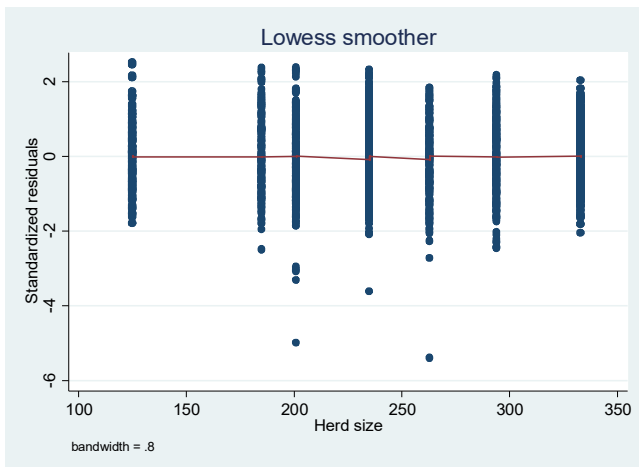


● Linearity (L1a - p11)

- ★ continuous predictors only
- ★ standardized residuals vs predictor

Herd size

Parity



- ★ also pattern can be evaluated by looking at the residuals from a model without the predictor of interest

# Evaluating individual observations

## Outliers (lecture L1a – pages 9/10)

- Large residuals
- Standardized residuals
  - ★ expect 5%  $> 2$  and  $< -2$
  - ★ expect 0.3%  $> 3$  and  $< -3$
- Deletion residuals
  - ★ t-test - prob =  $2 * n * t(DFE-1, d_i)$
- WPC log-transformed model – residuals
  - ★ standardized residuals
    - ➔ expected 5% = 79 (observed = 50)
    - ➔ expected 0.3% = 5 (observed = 6)
    - ➔ expected and observed values very similar
  - ★ deletion residuals
    - ➔ outlier cutoff value = 4.17
    - ➔ two observations  $\geq$  this cutoff point
      - indication that these are outliers
        - cows with wpc = 1
    - ➔ delete and refit the model (later)
      - only for diagnostics purposes

## Leverage

- X-outliers
  - ★ depend on X values only
    - not affected by relationship with y
  - ★ expresses whether  $x_i$  is outlying in distribution of x's
    - so potentially influential
  - ★ less meaningful for categorical predictors
    - depends on the distribution of each category
- Cut-off values
  - ★ concerns if  $h_i \geq 2(k+1)/n$ 
    - or  $\geq 3(k+1)/n$
    - $k = \#$  of predictors,  $n = \#$  obs.
    - should also look for unusual observations independently of these cutpoints
- Stata – *predict* varname, leverage
  - ★ eg. `predict lev_inwpc, lev`

## ● Computed after full model (ln\_wpc)

```
. summ lev
```

Variable	Obs	Mean	Std. Dev.	Min	Max
lev	1574	.0063532	.0083435	.0016636	.0642237

```
. display "leverage cutoff: " 2*nparam/nobs
```

**leverage cutoff: .01270648**

```
. display "conservative leverage cutoff: " 3*nparam/nobs
```

**conservative leverage cutoff: .01905972**

```
. count if lev>=.01905972 /* many (n=108) high leverage values */
```

```
. list wpc wpc_ln fit aut_calv herd_size parity1 twin dyst rp vag_disch stdres lev in 1/10, clean noobs
```

wpc	wpc_ln	fit	aut_calv	herd_s~e	parity1	twin	dyst	rp	vag_di~h	stdres	lev
76	4.331	4.690	yes	185	4	yes	no	yes	yes	-0.507	0.065
137	4.920	4.989	yes	294	2	yes	no	yes	yes	-0.097	0.064
94	4.543	4.859	yes	263	3	yes	no	yes	yes	-0.445	0.064
45	3.807	4.612	no	201	4	yes	no	yes	yes	-1.135	0.063
53	3.970	4.251	no	263	5	yes	no	no	yes	-0.395	0.059
110	4.700	4.188	no	263	0	yes	no	no	yes	0.720	0.057
32	3.466	4.286	no	201	1	yes	yes	yes	no	-1.150	0.052
99	4.595	4.575	yes	294	1	yes	yes	no	no	0.029	0.051
23	3.135	4.143	no	185	0	yes	yes	no	no	-1.409	0.050
106	4.663	4.600	no	294	0	no	yes	yes	yes	0.088	0.046

```
. table twin rp dyst
```

		Dystocia at calving and Retained placenta at calving			
		--- no ---		--- yes ---	
Twins	born	no	yes	no	yes
no	no	1332	124	76	15
yes	no	15	9	2	1

★ large leverage values for less common combination of categorical predictors

## Influence diagnostics: Cook's distance and DFITS

- Cook's distance and DFITS

$$\text{Cooks D} \quad D_i = \frac{r_{si}^2}{(k+1)} * \frac{h_i}{(1-h_i)}$$

$$\text{DFITS} \quad DFITS_i = r_{ti} \sqrt{\frac{h_i}{(1-h_i)}}$$

- ★ measure influence of an observation and have two interpretations:
  - effect of deleting observation “i” on the predictions
  - effect of outlier information about x's and y's
- ★  $D_i$  is on the squared residual scale and based on standardized residuals
- ★  $DFITS_i$  is on absolute (signed) residual scale and are based on deletion residuals

- cut-off values

- ★ concern if either is  $>1$  (rare) or if...

- $D_i \geq 1$  or  $\geq 4/n$

- $DFITS_i$  outside  $\pm 2 * \sqrt{\frac{k+1}{n}}$  (if  $n \geq 120$ )

- $n < 120$  only values beyond  $\pm 1$

- Stata: *predict* varname, cooksd/dfits

## ● Model: ln\_wpc

### ★ Cook's

```
. summ cook
```

Variable	Obs	Mean	Std. Dev.	Min	Max
-----+-----					
cook	1574	.0006351	.0013679	5.57e-10	.0174304

```
. display "Cook's D cutoff: " 4/nobs  
Cook's D cutoff: .0025413
```

```
. count if cook>=.0025413 /* many (n=94) high Cook's D values */  
94
```

### ★ DFITS

```
. summ dfit
```

Variable	Obs	Mean	Std. Dev.	Min	Max
-----+-----					
dfit	1574	.0003931	.0797594	-.4179343	.3664328

```
. display "DFITS cutoff: " 2*sqrt(nparam/nobs)* (nobs>=120)+1*(nobs<120)  
DFITS cutoff: .15941443
```

```
. count if abs(dfit)>=.15941443 /* many (n=92) high DFITS values */  
92
```

### ★ largest Cook's values

➔ not clear pattern – cows with at least one disease

### ★ largest negative DFIT values

➔ cows with twins and at least one of the diseases were the most influential

### ★ largest positive DFIT values

➔ cows without twins but with at least one of the diseases were the most influential

## Delta-beta (or DFBETA)

★ influence of an observation on a specific regression coefficient (  $\beta$  )

★ for obs. “i” and predictor  $x_j$

$$\rightarrow \text{DFBETA}_{ij} = \frac{(\beta_j - \beta_{j(i)})}{SE_j}$$

→ where  $\beta_{j(i)}$  is estimate of  $\beta$  without obs. “i”

★ measures change in  $\beta$  in terms of SE's

→ how many SE's does  $\beta_j$  increase(decrease) if “i” is dropped?

### ● cut-off values

★  $\text{DFBETA}_{ij} > 0$  ( $< 0$ ) ~ obs. “i” pulls  $\beta_j$  up (down)

→ extreme values of  $\text{DFBETA}_{ij}$  are notable

→ most meaningful for continuous predictors

★ concern if outside  $\pm \frac{2}{\sqrt{n}}$  (for  $n \geq 120$ )

→  $n < 120$  only values beyond +/- 1

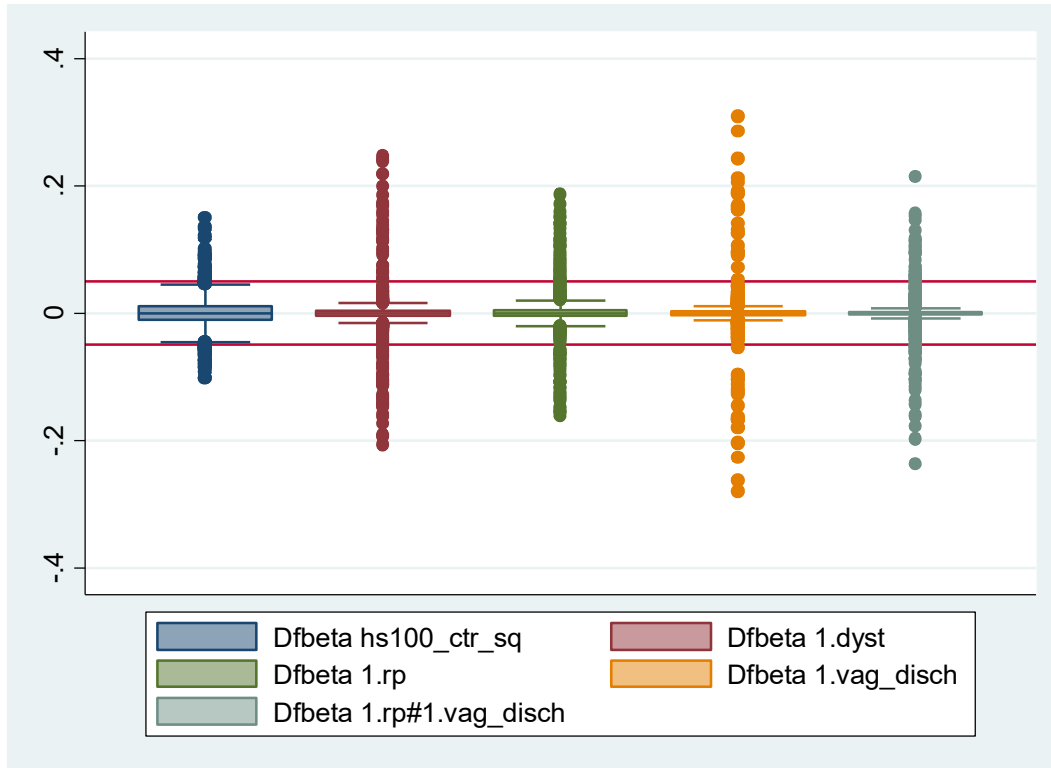
### ● Stata

★ *predict* varname, dfbeta(var in model)

→ eg. *predict* dfbdyst, dfbeta(dyst)

● Model: ln\_wpc

```
. display "DFBETA cutoff: " 2/sqrt(nobs)* (nobs>=120)+1*(nobs<120)  
DFBETA cutoff: .05041127
```



● Explore values by browsing/listing

- ★ large values are cases of the correspondent predictor (eg. cows with dystocia)
- ★ linear effect of herd size similar to quadratic term
  - ➔ need to be aware of collinearity

## What to do with Outliers/Influential observations

- Verify that point(s) (observation(s)) are not errors (eg. data entry errors)
- May be very informative
  - ★ always examine observations and values of the predictors
- Determine if outliers are significant (L1a -p10)
- Fit models with and without the outlier/influential observation(s)
  
- Keeping them in the analysis
  - ★ potential for biased estimates
  - ★ usually leads to larger SE and reduced power (is therefore conservative)
  - ★ always report them
- Eliminating them from the analysis
  - ★ **should be always reported and justified**
  - ★ narrows scope of inference from the study
  - ★ may create an unrealistically good model

## Other transformations

- log-transformed model

- ★ some indication of non-constant variance

- ★ residuals still not normally-distributed

- ➔ two severe outliers

```
. l wpc wpc_ln fit herd_size parity dyst rp vag_disch if wpc==1, clean compress
noobs
```

wpc	wpc~ln	fit	herd_size	parity	dyst	rp	vag_disch
1	0	3.645548	201	2	no	no	no
1	0	3.945861	263	2	no	no	no

```
. l wpc wpc_ln delres lev dfit cook dfb_dyst dfb_rp dfb_vd dfb_int dfb_hs if wpc==1, clean
compress noobs
```

wpc	wpc~ln	delres	lev	dfit	cook	dfb_dyst	dfb_rp	dfb_vd	dfb_int	dfb_hs
1	0	-5.028	0.002	-0.243	0.006	0.051	0.037	0.028	-0.021	0.098
1	0	-5.449	0.002	-0.243	0.006	0.044	0.053	0.042	-0.026	-0.016

- ★ comparison without outliers

```
. estimate table ln ln_noout
```

Variable	ln	ln_noout
hs100_ctr	.34365799	.33916323
hs100_ctr_sq	.21187276	.20269649
parity1	.01298436	.01133333
aut_calv	-.13716214	-.13695147
twin	.39274261	.3894441
dyst	.1109405	.10338895
rp		
yes	.11231661	.10603408
vag_disch		
yes	-.02601346	-.03328151
rp#vag_disch		
yes#yes	.41369992	.42230092
_cons	3.8885867	3.9010244

- ★ repeat but without influential obs.

- ★ advantage log-transformed model

- ➔ interpretation of estimates still possible

- Square root transformation

- ★ example VER – constant variance -> ok
- ★ residuals still not normally-distributed
- ★ deletion residuals show no indication of outlying observations
- ★ interpretation estimates not possible
  - ➔ obtain predicted values
  - ➔ parity=3 aut\_calv=1 hsize=0 hsize2=0 twin=0

rp#vag_disch#dyst	Original scale			Sqrt transformation			log-transformation		
	est	lo_ci	up_ci	est	lo_ci	up_ci	est	lo_ci	up_ci
no#no#no	<b>66.605</b>	62.220	70.990	<b>58.267</b>	54.583	62.071	<b>50.127</b>	47.005	53.456
no#no#yes	<b>78.305</b>	67.338	89.272	<b>66.840</b>	57.187	77.244	<b>56.008</b>	47.688	65.780
no#yes#no	<b>67.833</b>	53.767	81.899	<b>58.064</b>	46.695	70.671	<b>48.840</b>	39.737	60.027
no#yes#yes	<b>79.533</b>	63.311	95.755	<b>66.622</b>	52.636	82.255	<b>54.570</b>	43.017	69.225
yes#no#no	<b>72.592</b>	63.048	82.135	<b>64.364</b>	56.085	73.213	<b>56.085</b>	48.761	64.510
yes#no#yes	<b>84.292</b>	70.918	97.666	<b>73.359</b>	61.107	86.731	<b>62.666</b>	51.506	76.243
yes#yes#no	<b>96.672</b>	78.396	114.947	<b>90.259</b>	71.883	110.723	<b>82.645</b>	63.217	108.045
yes#yes#yes	<b>108.372</b>	87.603	129.141	<b>100.857</b>	78.877	125.534	<b>92.342</b>	68.098	125.216

\*ci estimated from t-distribution (Stata uses a different estimation procedure)

- ★ sqrt model – better than log-transformed
  - ➔ no outliers (eg. deletion res. range: -2.42; 3.28)
- ★ influential diagnostics
  - ➔ similar to log-transformed model
  - ➔ more in VER.