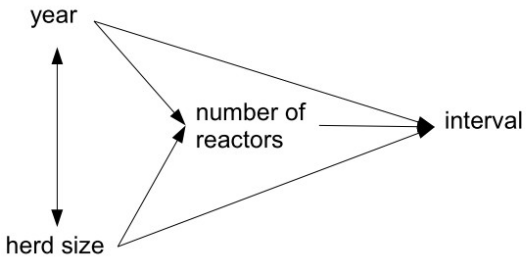
Linear Regression Exercise #2

Linear Regression – Model building

1. Draw a causal diagram showing how you think the three predictors might relate to the outcome and to each other.

Year and herd size are related because, in general, herds are getting larger over time. Whether this is presented as a causal relationship (one way arrow from year to herd size), or just as an association (two headed arrow) does not matter for this exercise. Both year and herd size may be related to number of reactors because the effectiveness of the control program maybe changing over the years and larger herds are more likely to have more reactors. All three factors may be related to the length of the interval between TB episodes.



2. Fit models with -p_year- as the only predictor. First fit it as a continuous variable and then as a categorical variable.

```
. regress intvl_ln p_year /* continuous predictor */

Source   |       SS       df       MS                 Number of obs =   3000
---------+------------------------------               F( 1, 2998) = 131.00
Model    | 147.547346     1   147.547346               Prob > F   =     0.0000
Residual | 3376.80672  2998   1.12635314               R-squared  =     0.0419
---------+------------------------------               Adj R-squared   =0.0415
Total    | 3524.35407  2999  1.17517641               Root MSE   =     1.0613

------------------------------------------------------------------------------
intvl_ln  |      Coef.   Std. Err.      t    P>|t|    [95% Conf. Interval]
----------+-------------------------------------------------------------------
 p_year   | -.0478439   .0041802   -11.45   0.000   -.0560402  -.0396475
_cons     |  102.3055   8.336632    12.27   0.000    85.95939   118.6516
------------------------------------------------------------------------------


. regress intvl_ln i.p_year /* categorical predictor */

      Source |       SS       df       MS              Number of obs =    3000
-------------+------------------------------           F( 18,  2981) =   13.21
       Model |  260.386099    18  14.4658944           Prob > F      =  0.0000
    Residual |  3263.96797  2981  1.09492384           R-squared     =  0.0739
-------------+------------------------------           Adj R-squared =  0.0683
       Total |  3524.35407  2999  1.17517641           Root MSE      =  1.0464

------------------------------------------------------------------------------
    intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      p_year |
       1990  |    .021196   .0788123     0.27   0.788     -.133336    .1757279
       1991  |  -.0087745   .0819222    -0.11   0.915    -.1694042    .1518552
       1992  |   .1245631   .0853603     1.46   0.145     -.042808    .2919342
       1993  |   .2724067    .094199     2.89   0.004      .087705    .4571084
       1994  |   .2170947   .1062763     2.04   0.041     .0087123    .4254771
       1995  |   .0149642   .1104147     0.14   0.892    -.2015325    .2314609
       1996  |   .1282697   .1058112     1.21   0.226    -.0792007    .3357402
```

```
        1997  |  -.0235158    .117306    -0.20   0.841    -.2535248    .2064932
        1998  |  -.3849102   .1184682    -3.25   0.001    -.6171979   -.1526225
        1999  |  -.3533293   .1046926    -3.37   0.001    -.5586064   -.1480521
        2000  |  -.2175166   .1131298    -1.92   0.055     -.439337    .0043037
        2001  |   -.146469   .1345969    -1.09   0.277    -.4103811    .1174432
        2002  |  -.4536966   .1307225    -3.47   0.001    -.7100121    -.197381
        2003  |  -.5163069   .1339179    -3.86   0.000    -.7788878    -.253726
        2004  |  -.6939645   .1509123    -4.60   0.000    -.9898673   -.3980616
        2005  |  -.9484242   .1696501    -5.59   0.000    -1.281067    -.615781
        2006  |  -1.380731   .1696501    -8.14   0.000    -1.713374   -1.048087
        2007  |  -1.910497   .6073476    -3.15   0.002     -3.10136   -.7196344
              |
        _cons |   6.957966   .0624222   111.47   0.000     6.835571    7.080361
------------------------------------------------------------------------------
```

(a) Interpret the two intercepts.

The intercept from the model with p_year as a continuous predictor is the estimated log-interval in the year 0 AD (i.e., the year Christ was born). Clearly this is an illogical value to try to estimate when all of the data come from 1989-2007. The intercept from the model with p_year as a categorical predictor is the average log-interval in 1989. This is a more reasonable thing to estimate.

(b) How could you improve the "interpretability" of the intercept from the first model? Make the necessary change(s) to do this.

You could center p_year by subtracting 1989 (i.e., observations with p_year=1989 will become p_year=0) so that the intercept becomes the estimated value for that year (7.144)

```
------------------------------------------------------------------------------
    intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  pyear_1989 |  -.0478439   .0041802   -11.45   0.000    -.0560402   -.0396475
       _cons |   7.144043   .0294498   242.58   0.000       7.0863    7.201787
------------------------------------------------------------------------------
```

(c) Which is better, the original model (with -p_year- as a continuous predictor) or this new model?

The model with -p_year- as a categorical variable has a large R^2, but this is achieved at a price. First, there are a lot of coefficients which are perhaps each of less interest. Second, there certainly appears to be a pattern (coefficients becoming increasingly negative), but the set of individual coefficients don't really describe that pattern). You could compute the F test statistic to compare these models (because the linear model is a submodel of the model using p_year as categorical).

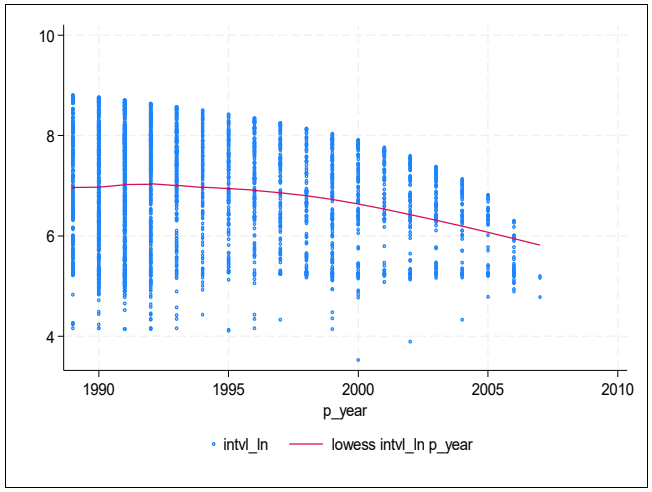
```
. scalar F=(3376.80672-3263.96797)/(2998-2981)/1.09492384

. display "lack-of-fit F-test:  F=" F "   P-value=" Ftail(2998-2981,2981,F)
lack-of-fit F-test:  F=6.0621326   P-value=4.551e-14
```

The tested null hypothesis (H_0) is that the simple model (p_year as continuous) shows the same fit as the more complex model (i.e., p_year as categorical). The very low p-value gives clear evidence against H_0 and therefore shows that the simple linear regression model doesn't fit the data as well as the categorical model. Perhaps the assumption of linearity inherent in the model with -p_year- as a continuous variable needs to be checked.

3. We need to determine if the relationship between -p_year- and intvl_ln- is linear.

(a) Evaluate this by fitting a scatter plot with a lowess smoothed curve through the points.



The scatterplot and lowess smoothed curve clearly show a non-linear relationship. The annual change (reduction) in the length of the interval was much greater after 2000 than before.

(b) Also evaluate it by fitting both linear and quadratic terms to the model.

```
. generate pyear_sq = p_year^2

. regress intvl_ln p_year pyear_sq

      Source |       SS           df       MS      Number of obs   =     3,000
-------------+----------------------------------   F(2, 2997)      =    100.97
       Model |  222.485518         2  111.242759   Prob > F        =    0.0000
    Residual |  3301.86855     2,997  1.10172457   R-squared       =    0.0631
-------------+----------------------------------   Adj R-squared   =    0.0625
       Total |  3524.35407     2,999  1.17517641   Root MSE        =    1.0496

------------------------------------------------------------------------------
    intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      p_year |   30.95684   3.759348     8.23   0.000     23.58568    38.32801
    pyear_sq |  -.0077659   .0009416    -8.25   0.000    -.0096122   -.0059196
       _cons |  -30843.28    3752.19    -8.22   0.000    -38200.41   -23486.16
------------------------------------------------------------------------------
```

The quadratic term is highly significant. It is also worth noting that the predictive ability of the model (as indicated by the R^2) has gone up substantially (from the model with the single linear term). Clearly, the quadratic model fits better than the linear one. Note that the coefficient for the linear term has changed completely, and is now impossible to interpret meaningfully on its own.

4. Compute the VIFs from the model with the quadratic term. Do these signal a problem with the model?

```
. estat vif
```

```
    Variable |       VIF       1/VIF
-------------+----------------------
      p_year | 826858.31    0.000001
    pyear_sq | 826858.31    0.000001
-------------+----------------------
    Mean VIF | 826858.31
```

The VIF values are huge (remember that anything over 10 is an indicator of substantial collinearity). Although the estimation and model fit don't seem to be affected of this, it is advisable to reparametrize to avoid a parameter (the intercept) being an extreme extrapolation.

(a) Explore how the VIFs change after centring -year- before squaring it. (Call the new variable pyear_ct-.)

Centring can be understood specifically as subtracting the mean value of the predictor, or less specifically as subtracting a "central" value of the predictor, such as the approximate mean or the median or some other value in the center of the predictor distribution. For -p_year-, the mean equals 1994.3 and the median equals 1993, so subtracting either 1994 or 1993 could work fine. Here will instead center by subtracting the midpoint (1998) of the range 1989-2007. Despite being a bit off the arithmetic mean, centring by this value will both improve the interpretation of the coefficients (the intercept now corresponding to 1998 as we aim for) and reduce the VIFs.

```
. generate pyear_ct=p_year-1998

. replace pyear_sq=pyear_ct^2
(3,000 real changes made)

. regress intvl_ln pyear_ct pyear_sq

      Source |       SS           df       MS      Number of obs   =     3,000
-------------+----------------------------------   F(2, 2997)      =    100.97
       Model |  222.485518         2  111.242759   Prob > F        =    0.0000
    Residual |  3301.86855     2,997  1.10172457   R-squared       =    0.0631
-------------+----------------------------------   Adj R-squared   =    0.0625
       Total |  3524.35407     2,999  1.17517641   Root MSE        =    1.0496

------------------------------------------------------------------------------
    intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    pyear_ct |  -.0757948   .0053458   -14.18   0.000    -.0862767    -.065313
    pyear_sq |  -.0077659   .0009416    -8.25   0.000    -.0096122   -.0059196
       _cons |   6.883048   .0319912   215.15   0.000     6.820321    6.945775
------------------------------------------------------------------------------

. estat vif

    Variable |       VIF       1/VIF
-------------+----------------------
    pyear_ct |      1.67    0.598086
    pyear_sq |      1.67    0.598086
-------------+----------------------
    Mean VIF |      1.67
```

Although the main purpose of centring is to improve the interpretability of the constant, here is helps to reduce the problem of collinearity between the linear and quadratic terms. Two things to note: 1) the linear term is now more reasonable (-0.076), and 2) the quadratic term is exactly the same in both models.

5. Is the year (fit as both linear and quadratic terms) a confounder for the effect of herd size (-hdsize-)?

```
. regress intvl_ln hdsize /* model without year */

      Source |       SS           df       MS      Number of obs   =     2,987
-------------+----------------------------------   F(1, 2985)      =      6.17
       Model |  7.24137996         1  7.24137996   Prob > F        =    0.0131
    Residual |  3504.58457     2,985  1.17406518   R-squared       =    0.0021
-------------+----------------------------------   Adj R-squared   =    0.0017
       Total |  3511.82595     2,986   1.1760971   Root MSE        =    1.0835

------------------------------------------------------------------------------
    intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      hdsize |  -.0010355    .000417    -2.48   0.013    -.0018531    -.000218
       _cons |   6.954334   .0324156   214.54   0.000     6.890774    7.017893
------------------------------------------------------------------------------

. regress intvl_ln hdsize pyear_ct pyear_sq /* model with year */

      Source |       SS           df       MS      Number of obs   =     2,987
-------------+----------------------------------   F(3, 2983)      =     67.71
       Model |  223.892476         3  74.6308254   Prob > F        =    0.0000
    Residual |  3287.93347     2,983  1.10222376   R-squared       =    0.0638
-------------+----------------------------------   Adj R-squared   =    0.0628
       Total |  3511.82595     2,986   1.1760971   Root MSE        =    1.0499

------------------------------------------------------------------------------
    intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      hdsize |  -.0002863    .000412    -0.69   0.487    -.0010941    .0005216
    pyear_ct |  -.0754918   .0053892   -14.01   0.000    -.0860586   -.0649249
    pyear_sq |  -.0078035   .0009457    -8.25   0.000    -.0096579   -.0059491
       _cons |   6.903749   .0433748   159.16   0.000     6.818701    6.988796
------------------------------------------------------------------------------

. display "percent change=" abs(-.0002863-(-.0010355))/abs(-.0010355)*100
percent change=72.351521

. * added assessment of association hdsize vs year (should really be adjusted for
herds)
. regress hdsize p_year

      Source |       SS           df       MS      Number of obs   =     2,987
-------------+----------------------------------   F(1, 2985)      =    113.81
       Model |  248014.759         1  248014.759   Prob > F        =    0.0000
    Residual |  6504857.89     2,985  2179.18187   R-squared       =    0.0367
-------------+----------------------------------   Adj R-squared   =    0.0364
       Total |  6752872.65     2,986  2261.51127   Root MSE        =    46.682

------------------------------------------------------------------------------
      hdsize |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      p_year |   1.965962   .1842821    10.67   0.000     1.604629    2.327295
       _cons |   -3859.25   367.5183   -10.50   0.000    -4579.865   -3138.635
```

Adding year to the model makes a substantial change in the estimate of the effect of herd size (it goes from -0.0010 to 0.0003, and becomes non-significant). Whether you consider year as a true confounder depends on whether you think that "year" is a cause of "herd size". Given that herds are getting larger over time, I think this is a reasonable way to view the relationship.

6. We want to evaluate an interaction, but to simplify this process, we are going to convert -p_year- into a dichotomous variable. There are many ways to do this, but the command that I used was....

```
. egen pyear_c2=cut(p_year), at(0 1999 2999) icodes
```

(a) Is there interaction between -p_rct- and this new dichotomous variable for year?

```
. regress intvl_ln pyear_c2##c.p_rct

      Source |       SS           df       MS      Number of obs   =     3,000
-------------+----------------------------------   F(3, 2996)      =     47.36
       Model |  159.565065         3   53.188355   Prob > F        =    0.0000
    Residual |    3364.789     2,996  1.12309379   R-squared       =    0.0453
-------------+----------------------------------   Adj R-squared   =    0.0443
       Total |  3524.35407     2,999  1.17517641   Root MSE        =    1.0598

------------------------------------------------------------------------------
       intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
----------------+-------------------------------------------------------------
     1.pyear_c2 |  -.4645391   .0577574    -8.04   0.000    -.5777874   -.3512909
          p_rct |   .0128656   .0036582     3.52   0.000     .0056927    .0200385
                |
pyear_c2#c.p_rct |
              1  |  -.0176227   .0081487    -2.16   0.031    -.0336003    -.001645
                |
          _cons |   6.963248   .0253014   275.21   0.000     6.913638    7.012858
------------------------------------------------------------------------------
```

Yes. The interaction term is weakly significant (P=0.031), suggesting that the effect of -p_rct- was different in the early years compared to the later years.

(b) What was the effect of p_rct in the early years (1989-1998)?

The predictive part of the model could be written as follows

$$Y = \beta_0 + \beta_1(pyear\_c2) + \beta_2(p\_rct) + \beta_3(pyear\_c2 * p\_rct)$$

When the coefficients are added, this becomes the following for early episodes:

$$Y = 6.96 - 0.465(0) + 0.013(p\_rct) - 0.017(0 * p\_rct) = 6.96 + 0.013(p\_rct)$$

which means that for each additional reactor, the log interval was expected to get 0.013 log-days longer.

(c) What was the effect of p_rct in the later years (1999 – 2007)?

For later episodes:

$$Y = 6.96 - 0.465(1) + 0.013(p\_rct) - 0.017(1 * p\_rct) = 6.495 + 0.013(p\_rct) - 0.017(p\_rct)$$
$$= 6.495 - 0.004\ (p\_rct)$$

which means that for each additional reactor, the log interval was expected to get 0.004 log-days shorter.

7. To simplify things, we will carry on using the dichotomous version of -p_year- (pyear_c2)

(a) If your main outcome of interest is p_rct, should either of the other two predictors be included in a regression model? Why?

Yes – they are both potential confounders. Also, -pyear_c2- must be included because it interacts with - p_rct-

(b) What is your final model? What do you conclude about the relationship between p_rct and intvl_ln?

```
. regress intvl_ln hdsize pyear_c2##c.p_rct, vsquish

      Source |       SS           df       MS      Number of obs   =     2,987
-------------+----------------------------------   F(4, 2982)      =     36.60
       Model |  164.348406         4  41.0871015   Prob > F        =    0.0000
    Residual |  3347.47754     2,982  1.12256122   R-squared       =    0.0468
-------------+----------------------------------   Adj R-squared   =    0.0455
       Total |  3511.82595     2,986   1.1760971   Root MSE        =    1.0595

------------------------------------------------------------------------------
      intvl_ln |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+--------------------------------------------------------------
        hdsize |   -.000811   .0004466    -1.82   0.070    -.0016867    .0000648
     1.pyear_c2 |  -.4590911   .0580284    -7.91   0.000    -.5728707   -.3453114
         p_rct |   .0149895   .0038641     3.88   0.000     .0074129    .0225661
pyear_c2#c.p_rct |
             1  |  -.0166078   .0081597    -2.04   0.042    -.0326071   -.0006085
         _cons |   7.003838   .0331564   211.24   0.000     6.938826     7.06885
------------------------------------------------------------------------------
```

The final model is shown above. The effect of -p_rct- is very similar to what is shown in question 6 so the calculations won't be repeated. The margins and marginplot commands were used to generate the graph below showing the relationship between p_rct- and –intvl_ln for the two categories of pyear_c2. Since there is one graph of each herd size the graph was created for a median herd size value (50).

```
. margins pyear_c2, at(hdsize=50 p_rct=(0(10)40)) expression(exp(predict(xb)))
. marginsplot
```